

Engineering the Trebuchet

From Ancient Siege Warfare to Optimal Design

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Abstract

Ancient and medieval writers credit stone-throwing engines with huge stones and long ranges. How far could the best possible trebuchet throw, and could it have broken a city wall? This paper answers with a physics model of the throw, checked against published models and measured reconstructions, and a design study that settles each feature of the machine by precedent and by calculation. Every cited work is graded for credibility in a sources register, and ancient and medieval numbers are treated as reports rather than measurements.

Built at a common size, with a 10 m beam and a 20 kg stone, the best machine in modern materials is a floating-arm trebuchet that throws 885 m in the model. The best that could have been built in 1187 is a tall hinged-counterweight engine with a tapered ash beam, which throws 580 m, or about 500 m allowing for the model's optimism. Medieval-pattern reconstructions reached 28 to 41% of that machine's range for their size. Their proportions were close to right for their frames, and the gap lies mostly in heavy beams and low frames, which suited builders who wanted heavy stones more than distance. Set against Saladin's siege of Jerusalem in 1187, the best buildable machine would have out-ranged the defenders' crossbows and struck with about the energy Josephus implies for Rome's engines in AD 70. On its own it would probably not have breached the wall. The sappers still made the breach.

A Note on Evidence and Units

Numbers from ancient and medieval authors are what those authors reported, not measurements, and the text says so where it uses them. Engines keep the names the sources give them, such as mangonel, petrary or *manjaniq*, because a chronicler's word is poor evidence of how a machine was powered.

Ranges and energies from our model are model values. Checked against real throws, the model runs about 10 to 15% long at laboratory scale, because it treats the beam, the frame and the sling as rigid. Comparisons between machines on equal terms are not affected. Results from the model carry run identifiers such as P7-R12 in the figure and table captions, which point to the records kept with the project.

Units are SI, with historical units in brackets. The cubit is taken as 0.457 m, the stade as 185 m and the talent as about 26 kg, close to the 25.8 kg of the Attic talent [1], [2], [3]. Dates are AD unless marked BC.

Companion Files

The sources register (Sources_Register.xlsx) lists every work consulted, its credibility grade and what was taken from it, with each fact the paper draws on. The full decision records, the two design specifications and the model runs are kept with it. The simulator described in Appendix

E, built with Claude [4], lets a reader build and throw both designs. Every figure was also created by Claude from the project's data, as its caption says.

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Prologue: “The Son Cometh”

In AD 70 the Roman army of Titus camped about two stades from the outermost wall of Jerusalem, opposite the corner tower called Psephinus [5, 5.133]. Every legion had brought engines for darts and stones, but those of the tenth legion, Josephus wrote, “were more forcible and larger than the rest” [5, 5.269]. Their stones “were of the weight of a talent, and were carried two furlongs and further” [5, 5.270]. That is about 26 kg thrown at least 370 m [2], [3].

The stones were white. The defenders could hear “the great noise” each one made and see it coming “by its brightness” [5, 5.271]. So watchmen sat on the towers, and whenever an engine let go they “cried out aloud, in their own country language, ‘The son cometh’” [5, 5.272], a cry that Whiston’s 1737 edition prints as “the stone cometh” [6, 5.6.3]. Anyone in the stone’s path threw himself flat [5, 5.272]. The Romans answered by blackening their stones. After that, Josephus says, they “destroyed many of them at one blow” [5, 5.273].

To place the engines, the Romans measured the distance to the wall “by lead and a line” thrown from their siege banks, and moved them up once they found the stones would carry that far [5, 5.275].

Josephus’s figures are reported, not measured, and Whiston’s two printings agree on them only because they are one translation [5, 5.270], [6, 5.6.3]. The ground has its own account. At the Russian Compound, west of the Old City, excavators found 79 ballista stones lying together in a patch 15 m by 5 m in front of the outermost wall [7, p. 109]. “Most of the stones weigh up to 5 kg, some weigh up to 11 kg, and one stone weighs 24 kg” [7, p. 110]. Arbiv puts the engines that threw them no more than 400 m away [7, p. 115]. One stone in 79 came near a talent.

How far could an engine like these throw, and how hard could its stone hit? Under both lies the question the watchmen lived with: could it break the wall? This paper puts those questions to the trebuchet, the beam-and-sling engine that came into its full power with the counterweight, first clearly recorded in 1165 [8, p. 113]. We design two of them. One is the best that physics and modern materials allow. The other is the best that could have been built from the timber and rope of 1187, the year Saladin’s mangonels came to Jerusalem. Then we ask

how close the machines people actually built came to either. The paper ends at Jerusalem, with the 1187 machine set before the wall that Saladin's sappers broke.

1. Before the Trebuchet

Sulla needed timber. In 87 or 86 BC his Roman army was besieging Athens and its port, the Piraeus, after the city sided with King Mithridates [9, p. 105], [10, 12.1]. Appian says he chopped down the grove of the Academy and built his largest engines there [11, p. 30]. Plutarch adds the Lyceum, and reports, in a suspiciously round number, that working the engines took ten thousand pairs of mules every day [10, 12.2–3]. Box 2 follows the siege to its end.

Sulla's engines belonged to a tradition three centuries old. Diodorus wrote that the catapult was invented at Syracuse, where Dionysius I had gathered the ablest craftsmen from everywhere to prepare for war with Carthage, and modern scholars put this at about 400 BC [12, 14.42.1], [13, p. 86], [14, p. 52]. By the 350s BC heavier machines were shooting stone balls, and Alexander mounted stone-throwers on his siege towers at Halicarnassus and Tyre [14, p. 53]. Their power came from springs of twisted sinew [15, 10.10.1]. Figure 1 sets these dates against the two thousand years of stone-throwing that followed.

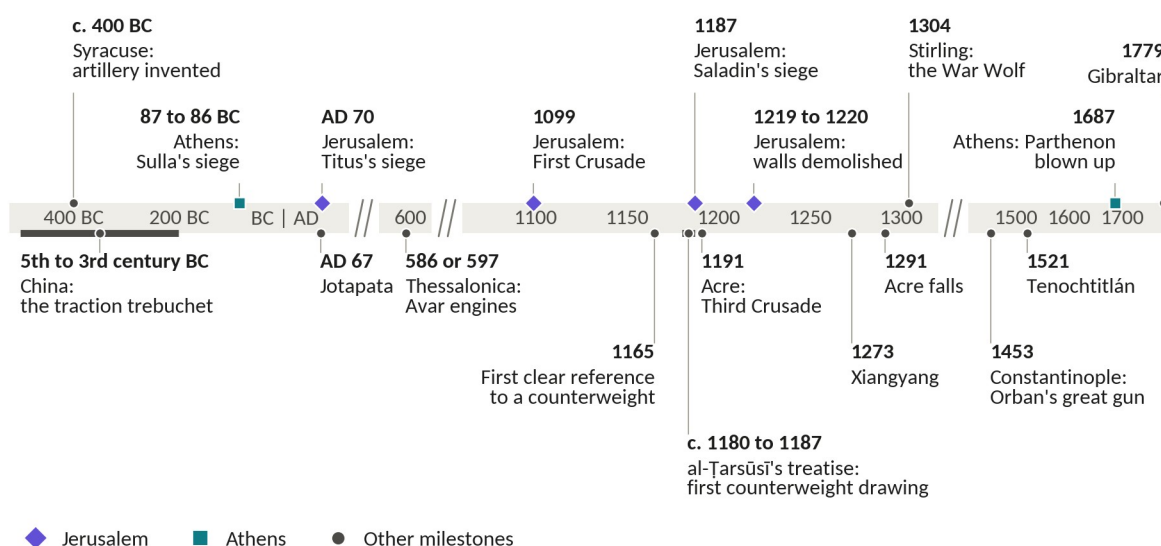


Figure 1. Two thousand years of stone-throwing engines, with the sieges of Jerusalem (violet diamonds) and Athens (teal squares) marked along the way. The axis is broken twice, and the years AD 1060 to 1320, when the counterweight engine appeared, are drawn 3.5 times larger than the rest. Dates from the sources cited in Chapters 1 to 8 and the Epilogue. Image created by Claude [4].

Greek engineers also learned to size their machines by arithmetic. Every part of an engine was a multiple of one measurement, the diameter of the hole that held the spring [16, p. 2]. For a stone-thrower, Heron's rule was to multiply the stone's weight in minas by a

hundred, take the cube root and add a tenth, which gave the hole in dactyls [14, p. 54]. In modern units the hole is about 0.13 m times the cube root of the stone's mass in kilograms [16, p. 3].

Box 1. The Catapult Formula with a Cube Root

By about 270 BC Greek engineers, perhaps working for the Ptolemies in Egypt, had turned catapult design into a formula [17]. Philo of Byzantium called the spring hole “the fundamental basis and unit of measure for the construction of engines”, to be found “by a standard method which could produce the correct proportions at all sizes” [16, p. 2]. For an arrow-shooter the hole was one-ninth of the arrow's length [15, 10.10.1]. For a stone-thrower it grew with the cube root of the stone's weight [17]. By the rule a stone of one talent, about 26 kg, needs a hole of 20 dactyls, roughly 0.39 m across, and a stone of three talents about 0.56 m [14, p. 54], [17].

Cube roots were hard to find. To double an engine's shot, Philo had to “double the cube”, since “the respective weights of the two missiles are in the proportion of the cubes of their diameters” [18]. Doubling the cube was the old Delian problem, and Philo attacked it through two mean proportionals [19]. The largest stone-thrower on record was a three-talent engine, for stones of about 78 kg, built by Archimedes [17]. Athenaeus puts it on Hiero's great ship [20, 5.208c].

The cube root carries a consequence for range. A spring of twisted cord stores energy in proportion to the cube of its diameter, and Heron's rule makes that cube grow in step with the weight of the stone [14, p. 54], [17]. Every kilogram of stone therefore gets the same share of energy, whatever the size of the engine, so a large engine built to the rule throws its heavy stone about as far as a small engine throws a light one. The rule buys weight, not distance. Vitruvius's table shows it at work, with eighteen calibres of a single design running from a 2-pound stone to a 360-pound one [15, 10.11.3]. Chapter 5 applies the same reasoning to the trebuchet, scoring each machine by its range divided by its size.

Ancient historians preferred drama to calibres. At Jotapata in AD 67, Josephus reports, Vespasian set 160 engines round the town, some of them throwing stones of a talent [5, 3.166–167]. One stone took off the head of a man standing beside Josephus on the wall and flung his skull three stades, more than half a kilometre [5, 3.245]. Josephus commanded the defence and wrote for effect, and a skull is not a stone.

Erwin Schramm's reconstructions, shown to the Kaiser at Metz in 1904, give the nearest thing to a measurement. They threw a one-pound lead ball and a four-pound ball 300 m each,

and an 88 cm bolt 370 m [13, pp. 85, 90–92]. If Schramm meant the German pound of 500 g, the balls weighed about 0.5 and 2 kg. Excavated shot is mostly light as well. At Rhodes most of 353 worked stones weighed 10 to 30 minas, about 4 to 13 kg, and 3,500 of the 5,600 balls examined at Carthage weighed 5.0 to 7.5 kg [14, pp. 56–58]. Philo of Byzantium himself judged that torsion engines could do walls little harm at 160 m [21, p. 67].

Box 2. Sulla Cuts Down the Academy

Plutarch tells the felling of the groves as a timber crisis. Many of Sulla's siege-works collapsed under their own weight or were burnt by the defenders' fire-bolts, so he "laid hands upon the sacred groves" and ravaged the Academy, "the most wooded of the city's suburbs" [10, 12.3]. Appian adds that Sulla pulled down the Long Walls and used "the stones, timber, and earth" for his siege mounds, while Thebes supplied iron and catapults [11, p. 30], [22, p. 30]. The war was paid for with the sacred treasures of Epidauros and Olympia, and Sulla sent to Delphi for more [10, 12.3–4].

Not everyone inside was loyal. Two slaves in the Piraeus, Appian says, wrote down everything that happened there, enclosed the writing in lead sling bullets and slung them over to the Romans [11, p. 31].

Sulla took Athens on the Calends of March, 1 March 86 BC [10, 14.6], [11, p. 38]. The date comes from his own memoirs [10, 14.6]. Plutarch leaned on those memoirs for much of the siege, so the story reaches us largely as the victor told it [9, p. 107].

Torsion stone-throwers were still on the walls of Rome in AD 537, when Belisarius set engines that "resemble slings and are called 'wild asses'" along the parapet [23, 5.21.18–19]. Within sixty years a different machine, with no spring at all, was hurling stones at Thessalonica [24].

2. From China to Jerusalem

Some time in the late sixth century an army of Avars and Slavs camped outside Thessalonica, and the city's bishop, John, described its engines [24], [25]. He saw four-sided frames, broad at the base and narrowing towards the top, where massive cylinders sheathed in iron carried "lengths of wood, like the beams of a large house, and these had slings" [8, p. 101], [24]. They threw large stones in frequent volleys, though John thought them inaccurate beside the city's own torsion artillery [24]. The year is disputed. Stephen McCotter follows Paul Lemerle in dating the siege to 586, and other scholars prefer 597 [8, p. 101], [24]. McCotter calls the passage "clearly a description of a trebuchet" [24]. Only the crews hauling on the ropes are missing, perhaps hidden from the bishop by sheaths of iron or leather [24].

This was the traction trebuchet. Chevedden and his co-authors trace it to China between the fifth and third centuries BC, and say it reached the eastern Mediterranean by the late 500s [21, pp. 66–67]. It had no spring. A beam turned on an axle, and a crew hauled on ropes tied to the short end so that the long end flew up and swung a sling [24], [26]. Crews of as many as 250 men are reported, sending projectiles 100 m or more [21, p. 68]. The later counterweight trebuchet replaced the crew with a heavily weighted box [24], and its earliest known drawing was made for Saladin (Box 3). Figure 2 names the parts and sketches the five ways of powering one.

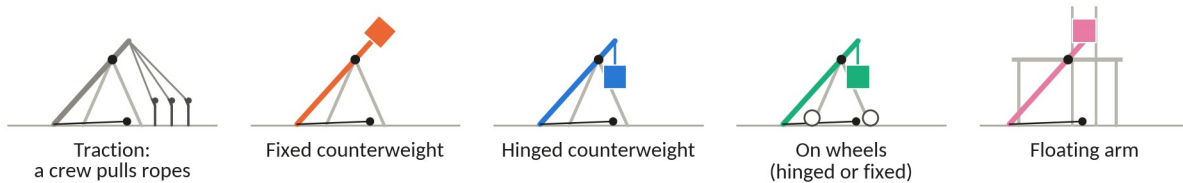
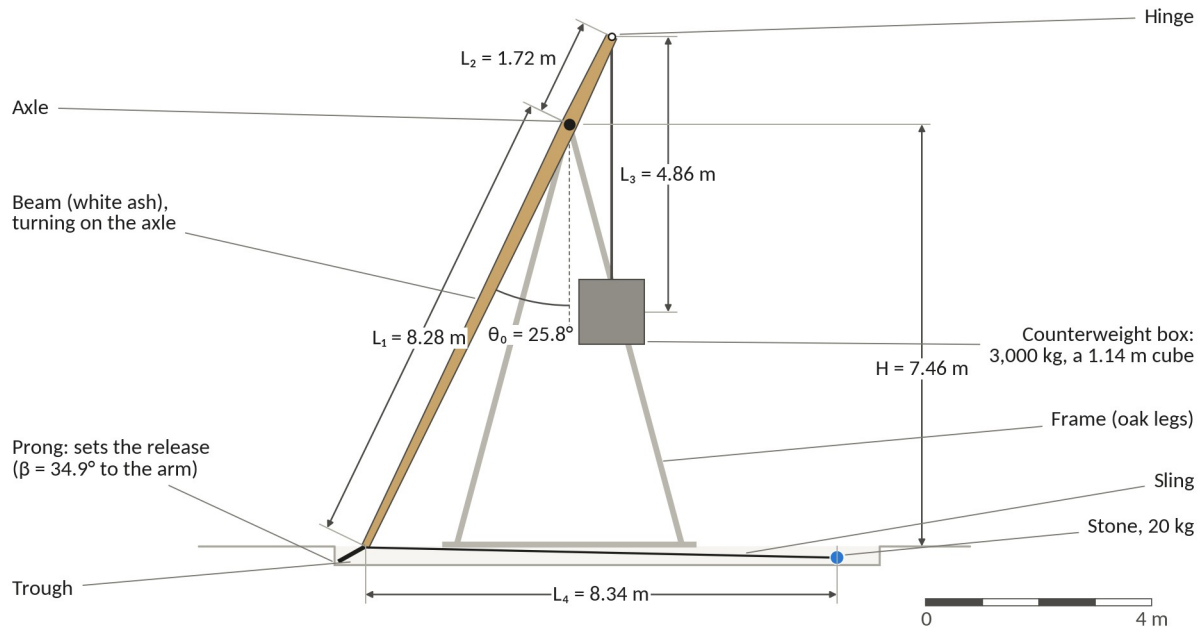


Figure 2. The parts of a trebuchet, and the five ways to power one. Top: the Jerusalem 1187 design of Chapter 6, cocked and drawn to scale, with the symbols used throughout the paper: long arm L_1 , short arm L_2 , hanger L_3 , sling L_4 , axle height H and starting angle θ_0 . Bottom, not to scale: traction, fixed counterweight, hinged counterweight, on wheels and floating arm. Source: run P7-R12. Image created by Claude [4].

Nobody yet had a name for it. In time it became the trebuchet in the West and the *helepolis*, “city-taker”, in Byzantium [8, p. 100]. Many writers used a word descended from the Greek *mánganon*, which first meant a love charm and went on to become the Arabic *manjanīq* and the English mangonel [24], [26], [27]. By 1155 Anglo-Norman verse paired mangonels with *perrieres*, or petraries [28], [29]. McCotter, following Bradbury, warns against deciding what a machine was from the word a chronicler used, so this paper keeps each source’s own term [24]. Even “trebuchet” is loose. The Anglo-Norman verb *trebucher* meant to hurl down or to overturn, and the noun could also mean a trap for small birds [30], [31].

Box 3. A Counterweight Engine Drawn for Saladin

The earliest known picture of a counterweight trebuchet comes from the court of the man who took Jerusalem in 1187. Murḍā ibn ‘Alī al-Ṭarsūsī wrote a military treatise, *Tabṣirat arbāb al-albāb*, “on the demand of Sultan Salah el-Din”, around 1187 according to Zaky’s bibliography of 1965 [32, pp. 107–9]. Among its chapters is one on “The Mongonel”, beside others on battering rams and siege towers [32, pp. 107–9]. Michael Fulton dates the work to about 1180. It must predate Saladin’s death in 1193, since it is dedicated to him [26].

Its drawing remains, in Peter Purton’s words, “the first known illustration of the counterweight engine”. Fulton judges that “the immaturity of the design is clear”, unlike the other engines al-Ṭarsūsī drew [26]. A manuscript of the treatise survives in the Bodleian Library in Oxford as MS. Huntington 264, catalogued as a treatise on weaponry written for Saladin [32, pp. 107–9], [33]. The drawing shows that Saladin’s circle knew the machine. It does not show that his army used one at Jerusalem.

Jerusalem fell to the First Crusade after a siege of 38 days, from 7 June to 15 July 1099 [34, Ch. 27], [35, p. 262]. Wood for siege towers had to be fetched from a distance [35, pp. 256–57]. Genoese sailors under William Embriaco, who had lost their ships at Jaffa, saved “ropes, mallets, spikes, axes, and hatchets” for Count Raymond’s works [36, pp. 257–62]. A modern account has the ships themselves broken up and their timber carried inland [37, p. 134], but the eyewitness chronicler Raymond of Aguilers mentions only the tools.

Artillery did not take the city. Rams broke a hole through one part of the wall, and the defenders hung beams on ropes in front of it [34, Ch. 27]. On the night before the last assault, Duke Godfrey and the counts of Flanders and Normandy had their machines carried in pieces almost a mile, to the stretch between the church of St Stephen and the valley of Josaphat [36, pp. 257–62]. The *Gesta Francorum*, another eyewitness account, calls it the eastern side [35, pp. 256–57]. Siege towers reached the wall on 13 and 14 July [38]. On 15 July the drawbridge of a wooden tower, hinged at its middle, swung down onto the wall to make a bridge into the city [36, pp. 257–62]. Godfrey’s men took a sector of the wall, and others followed on scaling ladders [38]. “Jerusalem was taken from the north”, wrote Ibn al-Athīr [39]. That northern stretch of wall will matter again in Chapter 8. Box 4 tells how the artillery fared.

Box 4. Nine Engines to One, and Two Would-Be Witches

The artillery fight for Jerusalem was not one-sided. Seeing how many engines the crusaders had built, the defenders strengthened the weak parts of their wall, and the Gesta says they raised their towers by night [35, pp. 256–57], [36, pp. 257–62]. Once the assault began, wrote Raymond of Aguilers, “from every side stones were hurled from the *tormenti* and the *petrahae*, and so many arrows that they fell like hail” [36, pp. 257–62]. Firebrands flew too. “The wood was dipped in pitch, wax, and sulphur”, and straw and tow were bound on with an iron band [36, pp. 257–62].

On the last morning, Raymond claimed, the defenders had nine or ten machines for every one of the crusaders’, and “our machines were now shaken apart by the blows of many stones” [36, pp. 257–62]. The ratio is almost certainly inflated. He paused for one strange incident. Two women tried to bewitch one of the hurling machines, but a stone struck and crushed them, together with three slaves [36, pp. 257–62]. The English translation does not say whose machine it was, or whose stone.

Raymond never says how the engines of 1099 were powered [36, pp. 257–62]. When the counterweight arrived is disputed. George Dennis reads the winch in an account of Andronikos Komnenos at Zevgminon in 1165 as the first clear reference to a counterweight machine, since a traction engine needs no winch [8, p. 113]. Michael Fulton puts it in the last two decades of the twelfth century, as a heavier engine that supplemented the light traction machines [26], [40]. He rejects the “hybrid”, a traction engine with a weight added to its short arm, as mechanically improbable [26]. The first known drawing of one is in al-Ṭarsūsī’s treatise for Saladin, dated to about 1180 by Fulton and to about 1187 by Zaky (Box 3) [26], [32, p. 107].

None of the sources read for this paper says that counterweight engines were used at Jerusalem in 1187. Chapter 8 therefore calls Saladin’s machines mangonels or engines, as the translated chronicles do.

Over the next century the engines grew larger, and their builders travelled. In 1271 Kublai Khan sent to the Ilkhan in Persia for engine-makers, and two Muslim engineers, ‘Alā’ al-Dīn and Ismā‘īl, came to China [41]. According to the official Yuan history, Ismā‘īl’s engine at Xiangyang in 1273 threw a stone of 150 *jīn* that sank seven *chǐ* into the ground [41]. At the Mamluk capture of Acre in 1291, Abu’l-Fida, who later wrote the episode down, had charge of one cart of a dismantled engine. The whole engine filled a hundred [42, 2:209], [43].

3. Building and Firing

Edward I had been besieging Stirling Castle since 21 April 1304 when he gave an unusual order. None of his men, he said, were to enter the castle until it had been struck by his new engine, the “war wolf” [44, pp. 180–181], [45, xliii]. Box 5 tells the story.

A siege engine was a building before it was a weapon. Villard de Honnecourt’s early thirteenth-century sketchbook gives the base frame of a trebuchet and, for its counterweight, a chest full of earth “two great toises long, nine feet wide and twelve feet deep”, about 3.9 by 2.9 by 3.9 m [46, Sec. 7.2], [47, entry “Engin”]. Al-Ṭarsūsī preferred cherry for the beam, or failing that a close-grained wood such as cedar, and held that a beam flexible “but not excessively so” throws farther [48, p. 149]. English soldiers under Edward I greased the axle with lard [49, p. 72].

Box 5. Edward I Waits for His War Wolf

To supply “the weights of his siege-engines” before the siege of Stirling, Edward ordered the Prince of Wales to strip lead from roofs, sparing churches only the parts above the altars [44, p. 180], [45, xlii]. Twenty-two carts of lead came from St Andrews alone [50]. He had done this before. In 1303 he stripped the lead off Brechin cathedral for his engines, though he restored it afterwards [45, xl]. Besides the thirteen engines counted by the chronicler Pierre de Langtoft, the king had a new one “of novel construction” [44, p. 181]. The royal records call it “le Lup de guerre”, and Langtoft’s French verse calls it “Ludgar” [51].

When War Wolf was nearly built, the garrison offered to surrender. Edward refused, says Historic Environment Scotland, so that he could make a public spectacle of his power [50]. Langtoft, who was not there, wrote that “at its stroke” it “broke down the entire wall” [44, p. 181]. The castle surrendered on St Margaret’s Day, 20 July 1304 [44, p. 181]. The order to strip lead “for the weights” is documentary evidence that some of Edward’s engines had counterweights.

Engines needed carpenters, and carpenters were paid. At the siege of Nottingham in 1194 the Pipe Roll records payment to “Master Roger the carpenter and his fellows” for their machines, among them at least one petrary and one mangonel [52]. In the twelfth-century Near East, Latin armies generally built their artillery on the spot from local timber rather than prefabricating it [53, p. 15]. A century later the English shipped theirs. The engines for Caerlaverock in 1300 came with the fleet, and in 1303 Edward I brought his by sea to Montrose

[45, xl], [54, xi]. In 1335 Guido da Vigevano advised the French king to prefabricate siege equipment in parts small enough to carry on horseback [55].

The counterweight changed the arithmetic of labour. The largest traction engines are reported to have needed crews of up to 250 (Chapter 2), while Hansen's counterweight replica in Box 6 needs ten or twelve people, all but two of them to haul its arm down [21, p. 68], [49, p. 75], [56]. The saving cost time. A small traction machine worked by six pullers can loose a stone about every 20 s, and the Danish replica fires about once every ten minutes [25], [56].

What did the engines throw? Excavation gives a measured answer. Eight shots found outside the walls of Kenilworth Castle, from the siege of 1266, weigh from 1 to 105 kg [57]. Of 2,747 stones from the Mamluk siege of Arsuf in 1265, 800 were weighed, and most of those came to 8 to 48 kg [58, p. 87]. The heaviest reached 78 kg, and only 11 passed 70 kg [58, p. 87]. The stone missiles found at Shawbak average about 60 kg [59, p. 31]. Against these weighed stones stand a modern estimate of 200 to 300 kg for the shot of a large trebuchet, and reports, naming no machine, of a few that threw 900 to 1,360 kg [21, p. 68].

Stones were not the only load. At Caffa in the Crimea, besieged in 1345 and 1346, Gabriele de' Mussi wrote that the dying besiegers had corpses lobbed into the city [60]. Mark Wheelis doubts that de' Mussi was there, and judges that Caffa had no more than anecdotal importance in bringing plague to Europe [60]. Modern builders have thrown stranger things, for fun. Hew Kennedy, who built a trebuchet on his Shropshire estate around 1990, threw upright and grand pianos with it, and dead farm animals [61], [62].

The engines could be aimed as well. At Vincennes in 1850 no shot strayed 3 m sideways, and three shots with one stone from Tanel Saimre's machine at Otepää landed within 2 m of each other [49, p. 77], [63, p. 35]. Box 6 describes the Danish machine, which kept its line over about 120 shots [56].

Box 6. The Danish Trebuchet That Kept Its Aim

In the summer of 1989 Peter Vemming Hansen led the building of a full-size trebuchet at the Medieval Centre in Nykøbing Falster [56], [64]. His team included a smith, a millwright and a rigger [49, pp. 75–76]. They made models at 1:10 and 1:5 before building the real machine, and took its base from a thirteenth-century ground plan by Villard de Honnecourt and the rest from Viollet-le-Duc's drawings of 1854 [49, pp. 75–76].

The machine was built for 4,000 kg of ballast but had been tried only with 2,000 kg, which threw a 15 kg cast-cement ball 168 m [49, pp. 75–76], [56]. Each extra 500 kg in the box added about 30 m [56]. What struck Hansen most was the aim. His shots held their direction even at long range, and none strayed more than 2 to 3 m to either side of the line [56]. Scientific American later reported that a medium-sized machine from the Falsters Minder museum in Denmark grouped its shots inside a 6 m square at 180 m [21, p. 69]. It does not say whether this was Hansen's machine.

Table 1 collects what reconstructions have thrown, keeping measured shots apart from reported ones. Hansen's replica is the best documented, with 168 m from a 15 kg ball [56]. Saimre's Otepää machine sent a 5 kg stone 120 m, but from a hillfort 35 m above the surrounding ground, and he put the level-ground equivalent at 85 m or less [49, p. 77]. Measured traction engines reached 80 to 145 m, with stones of 2 to 4 kg [25], [48, p. 162]. One column of the table gives R^* , the range divided by the length of the beam. Chapter 5 explains why that size-free score is the fair way to compare machines of different sizes.

Table 1. What reconstructions actually threw. Ranges are best shots unless marked. R^* is the reported range divided by the beam length ($L_1 + L_2$), shown where run P8-R01 computed it.

Machine and year	Counterweight (kg)	Stone (kg)	Range (m)	R^* (reported)	How measured	Source
Favé's trebuchet, Vincennes, 1850	4,500	11.7 (24 livres)	175	n/a	Trial report	[63, pp. 34–35]
Hansen's replica, Nykøbing Falster, 1989	2,000	15	168	23.8	Measured, best shot	[56]
Lihula, Estonia, 1997	300	5	about 40	n/a	Published trial, at second hand	[49, p. 75]
Lihula, Estonia, 1997	600	8	100	n/a	Published trial, at second hand	[49, p. 75]
Varbola, Estonia, 1999	1,400	10	120	16.0	Builders' own claim	[49, pp. 75–76]
Saimre's machine, Otepää, 2002	250	5	120 (85 on level ground)	n/a	Measured, from a 35 m hill	[49, p. 77]
Tarver's traction	none (14 pullers)	3.1	145	29.7	Measured, best	[48, p.

Machine and year	Counterweight (kg)	Stone (kg)	Range (m)	R* (reported)	How measured	Source
machine, 1991					shot	162]
NTNU traction machines, Norway	none (6 pullers)	2 to 4	about 80 (at most 120)	about 12	Measured, typical	[25]
A reconstruction in England, reported 1995	"30-ton"	476 (a car)	80	n/a	Reported	[21, p. 68]

4. What Worked

In 1850 a trebuchet with a 4,500 kg counterweight was tried at the Vincennes artillery range near Paris [63, pp. 34–35]. The weight was not the problem. The short end of its 10.3 m arm was only 30 cm long, so the counterweight dropped about half a metre and had little energy to give [49, p. 74]. Box 7 tells how the trials ended.

Box 7. The Prince-President's Trebuchet

The Vincennes machine was built on the order of Louis-Napoléon Bonaparte, then President of the French Republic, and Captain Favé ran the trials [63, pp. 34–35]. Louis-Napoléon designed the sling himself and specified the curve and mounting of its iron tip [56]. Of the 4,500 kg counterweight, 1,500 kg was fixed to the arm and 3,000 kg hung free [49, pp. 74–75].

Favé reported a ball of 24 livres, about 11.7 kg, thrown 175 m, and an earth-filled shell 0.22 m across thrown 145 m [63, p. 35]. Two larger shells went 120 m. The aim was good, and no shot strayed as much as 3 m sideways [63, p. 35]. The range was poor. Hansen's Danish replica of 1989 (Box 6), with less than half the counterweight, threw a heavier ball, 15 kg, almost as far, 168 m [56]. Favé's machine fired four times before its supporting structure gave way, and the experiment had to stop [49, pp. 74–75]. No plans of it survive, so its proportions are known only from these reports [56].

Much depended on the sling. It was fixed to the beam at one end and looped over a metal prong at the other, and at the right point in the swing the loop flew off and let the stone go [21, p. 68]. The prong decided that moment [49, p. 72]. Counterweight engines carried much longer slings than traction engines, and a trough under the beam held the stone at the start so that the sling could be longer still [21, p. 68]. Dufour's nineteenth-century experiments, reported in Yule's notes to Marco Polo [42, p. 207], showed that without a sling the range would fall by more than half, and on a modern bench engine a sling threw 32 m where a cup threw 5.9 m [65, table I].

Chevedden and his co-authors write that medieval engineers found a hinged counterweight, hanging from the beam, threw farther than one fixed to it [21, pp. 68–69]. The medieval voice on the question is Giles of Rome, who sorted stone-throwing engines into four kinds. His *trabutium* had its weight fixed to the beam and shot more regularly, because the weight always acted the same way, while his *biffa* let the weight swing about the beam, which

gave more force but less regular shooting [63, p. 29]. A third kind carried both weights, and the fourth was pulled by men, throwing smaller stones but ready to shoot sooner [63, p. 29].

Few medieval proportions survive. Marino Sanudo is said to have thought the best ratio of long arm to short was 5.5 or 6 to 1, but we know this only through Hansen and Saimre [49, p. 68]. A drawing in Conrad Kyeser's *Bellifortis* gives a long arm of 46 feet and a short arm of 8, a ratio of 5.75 [21, p. 71]. Modern builders work from rules of thumb, and Saimre, partly after Siano, gives four of them: an arm ratio of 4 to 5, a sling about as long as the long arm, a counterweight about 100 times the stone and a hanger at least half the short arm [49, pp. 70–71, 75]. Crews aimed with the sling and the release. A bigger stone needed a shorter sling, and on Hansen's machine each wooden ring slipped over the release hook took 5 m off the range [49, pp. 71–72]. Table 2 sets each rule beside a modern optimum and the choices made in Chapter 6.

How good could a trebuchet be? Erik Horsdal found an apparent consensus in the literature that the stone could receive at most about 70% of the energy the counterweight gave up [46, Sec. 6.3]. His own optimised hinged designs put more than 90% into the stone, in calculations without friction or air [46, Sec. 6.3].

Table 2. What the builders knew, beside a modern optimum and the two designs of Chapter 6. Horsdal's figures are for his reference design [46, table 1], whose R^* is 51, and our designs' figures come from run P7-R12. The last column points to the decision records in Appendix D.

Rule or feature	Precedent	Horsdal's optimum	Jerusalem 1187	Optimal	Decision record
Hinged counterweight	Said to throw farther [21, pp. 68–69]. Giles of Rome: a fixed weight shoots more regularly [63, p. 29]	Hinged	Hinged	Floating arm (weight drops in guides)	DR-01, DR-03
Arm ratio L_1/L_2	5.5 to 6, Sanudo at second hand [49, p. 68]. 5.75, Kyeser [21, p. 71]. 4 to 5, modern rule [49, p. 75]	4.9	4.8	3.1	DR-04
Sling ratio L_4/L_1	About 1 [49, p. 71]	0.93	1.0	1.1	DR-11
Counterweight ratio M/m	About 100 [49, p. 71]	105	150	150	DR-09
Hanger ratio L_3/L_2	At least 0.5 [49, p. 70]	Long: $L_2 + L_3$ from 0.5 to 1 times the axle	2.8	No hanger	DR-03

Rule or feature	Precedent	Horsdal's optimum	Jerusalem 1187	Optimal	Decision record
A sling, not a cup	More than doubles the range, Dufour in Yule [42, p. 207]	height Sling	Hemp rope, 25 mm	Synthetic (HMPE) rope, 8 mm	DR-12
Aim by the release	5 m less per ring on the hook [49, p. 72]	n/a	Iron prong, bent to tune	Steel prong, adjustable by up to 10° either way	DR-13
Efficiency η	About 0.70, the literature's apparent limit [46, Sec. 6.3]	0.90 to 0.93	0.75	0.91	DR-01

Modern machines go much farther. The competition record, 1,029 m with a pumpkin of 3.6 to 4.5 kg (8 to 10 lb), was set in 2019 by Colossal Thunder, a “whipper” whose frame rolls on tracks [66], [67], [68], [69, p. 3]. Yankee Siege, a pumpkin thrower in New Hampshire, went farther each time its arm was lightened, reaching 620 m in 2009 (Box 9) [70]. The floating arm, whose weight drops straight down in guides, beat the hinged machine in the only quantified comparison found, by 0.83 to 0.81 in range efficiency, and that study survives only as an abstract [71]. The case for putting a trebuchet on wheels rests on a scale model shown in a television documentary [72]. Chapter 6 tests each of these beliefs, from the hinge to the length of the sling.

5. The Physics of a Throw

Horsdal's Villard-inspired design hangs 19.1 tonnes of counterweight from a 7.34 m beam to throw a 100 kg stone [73, table 1]. By his calculation the stone leaves at 66 m/s carrying 91% of the energy available, and for an instant the beam holds almost 80 kJ on its way to the stone [73, Secs. 10.4, 10.6, 11]. Our model, written independently, reproduces his range and his peak axle load to within 0.2%.

The model treats a trebuchet as stiff parts joined by pins, and Lagrange's method, a standard procedure of mechanics, turns their energies into equations of motion (Appendix A).

A throw has four stages (Figure 3). The stone slides along a trough while the arm begins to turn, then lifts off and swings out in the sling, which whips it round faster than the arm turns. The counterweight, free on its hinge, falls almost straight down. A weight bolted to the beam must swing round an arc and keeps the energy of that sideways motion, so the hinge wins. Release comes when a ring on the sling's free end slides off a prong at the arm's tip, so the prong's angle sets the moment [21, p. 68], [74, Sec. 2.1]. Then the stone flies, slowed by the air.

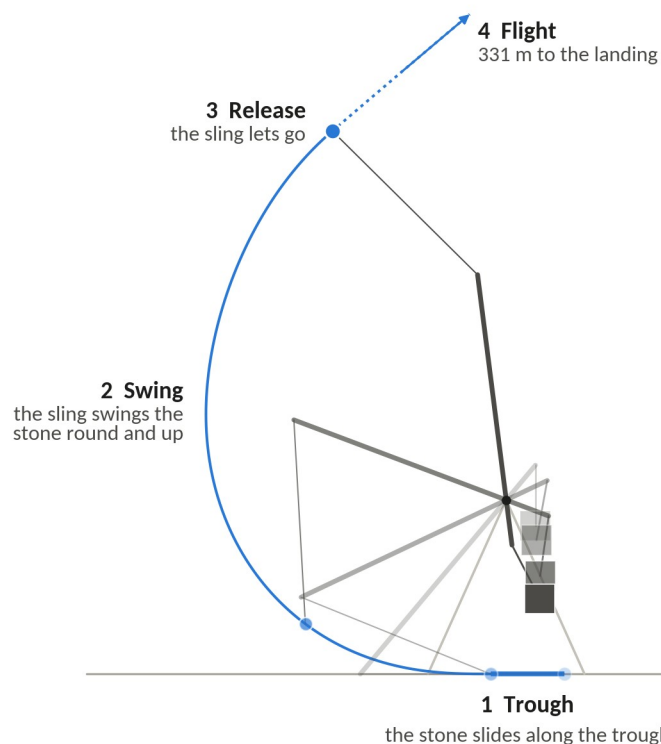


Figure 3. A throw in four stages: the stone slides along the trough, swings round and up in the sling, leaves when the sling lets go 1.12 s after the trigger, and flies 331 m. The machine is an illustrative hinged-counterweight trebuchet (6 m long arm, 8,000 kg counterweight, 60 kg stone), not an optimised design, drawn at the trigger, at lift-off, in mid-swing and at release. Source: model run P6-R14. Image created by Claude [4].

Figure 4 follows the energy through one throw of the laboratory engine Horsdal and his colleagues tested at Aarhus. The falling counterweight feeds the beam, which passes most of it on to the stone. Friction and air take about 5% [74, Sec. 3.5]. Whatever the stone does not carry away stays in the swinging machine. We score a throw by its efficiency,

the stone's energy of motion at release (mass m , speed v) divided by the energy the counterweight (mass M) could give up by falling its full drop Δh . Horsdal's looser measure also credits the stone's height and leaves out the energy spent lifting the beam, so his 91% design scores 0.76 on ours.

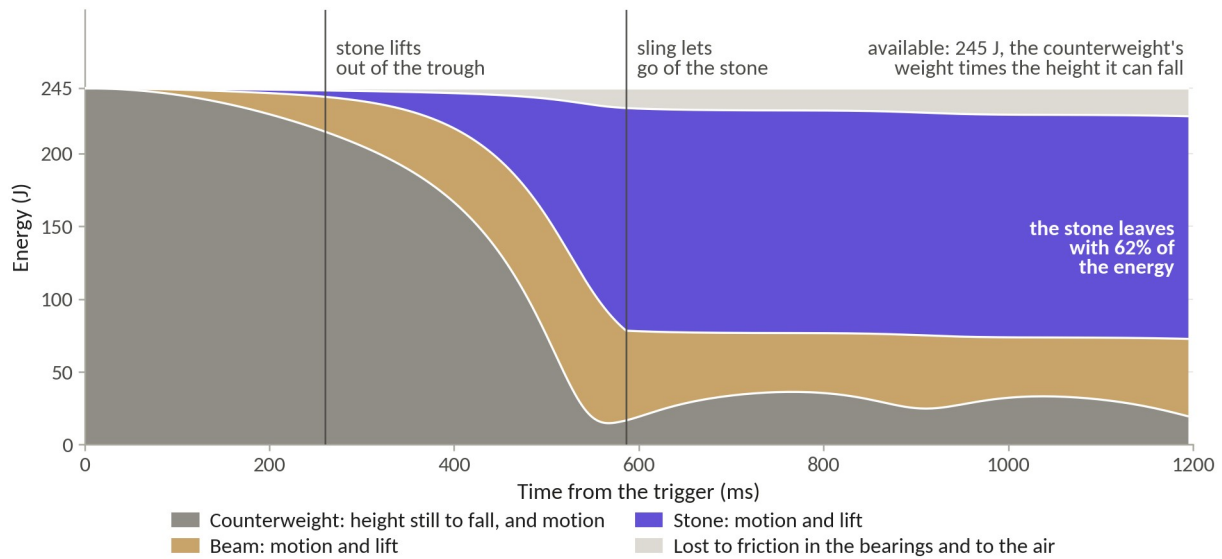


Figure 4. The counterweight's energy passes through the beam into the stone, and friction and air take only a thin slice: on the Aarhus laboratory engine the stone leaves with 62% of the 245 J available, counting its height as well as its speed. The bands show where the energy sits from the trigger to 1.2 s. The stone lifts out of the trough at 260 ms and the sling lets go at 586 ms. Source: model run P6-R03 (0.70 kg stone, 0.87 m sling), re-run for this figure. Image created by Claude [4].

Bearing friction grows with the load on the pin, so the model recomputes it at every instant [75, eq. 3]. Drag is less certain. Stones of this size fly near the “drag crisis”, the speed at which the drag on a smooth sphere suddenly collapses, and roughness shifts that point [76, eq. 1], [77]. We take a drag coefficient of 0.40, the value for rough stone beyond the crisis. For both designs the energy books balance to one part in a billion.

A Score Free of Size

Built to the same proportions, a trebuchet twice the size throws twice as far in a vacuum, because its equations of motion, written in proportions, do not depend on size [75, Sec. 6.1]. Horsdal's scaled engines from 1.2 m to 10 m all throw 33.7 times the length of their beam [75, Secs. 6.1, 6.7, table 6]. So we score a machine by its range R divided by its beam's length, long arm L_1 plus short arm L_2 :

Philo of Byzantium, as Chapter 1 showed, sized every part of a torsion catapult from one module so that its proportions held at all sizes [16, p. 2], and R^* rests on the same idea. Box 8 says what the number means. Strength does not scale: twice the size puts twice the stress in the same beam, so Chapter 6 judges materials at one reference size.

Box 8. What R^* Means

R^* is the range divided by the length of the beam. It asks how far a machine throws for its size, so a model and a full-size engine built to the same proportions, with the counterweight the same multiple of the stone, earn the same score. Shrink Horsdal's 19.1 t design to a quarter of its size and it throws 112 m from a 1.8 m beam. Double it and it throws 900 m from a 14.7 m beam. In a vacuum both score 61.3.

How We Know It Works

The model reproduces every published model we could set up completely, mostly to within 1%. Its friction losses for the Aarhus engine come within 3% of Horsdal's [75, table 2], and it matches de Jong's models of the Danish machine to 0.1 m and Levert's to 0.2% [78, pp. 12, 26–27], [79, pp. 25–28].

Real throws are harder (Figure 5). The model puts the Aarhus engine's field throws 9 to 17% long [74, table 3], [75, Sec. 5.4]. Horsdal measured 33 J lost per throw, more than twice what friction and drag explain, and traced the rest to motions no rigid model includes, such as the trestle shifting and the beam bending [75, Sec. 5.3]. Hansen's Danish machine threw a 15 kg ball 168 m [56, "The Trials"]. With central guesses for its unweighed beam the model gives 180 m, and across the plausible range 111 to 253 m. Our ranges are therefore model values, about 10 to 15% optimistic at laboratory scale, a bias every design shares.

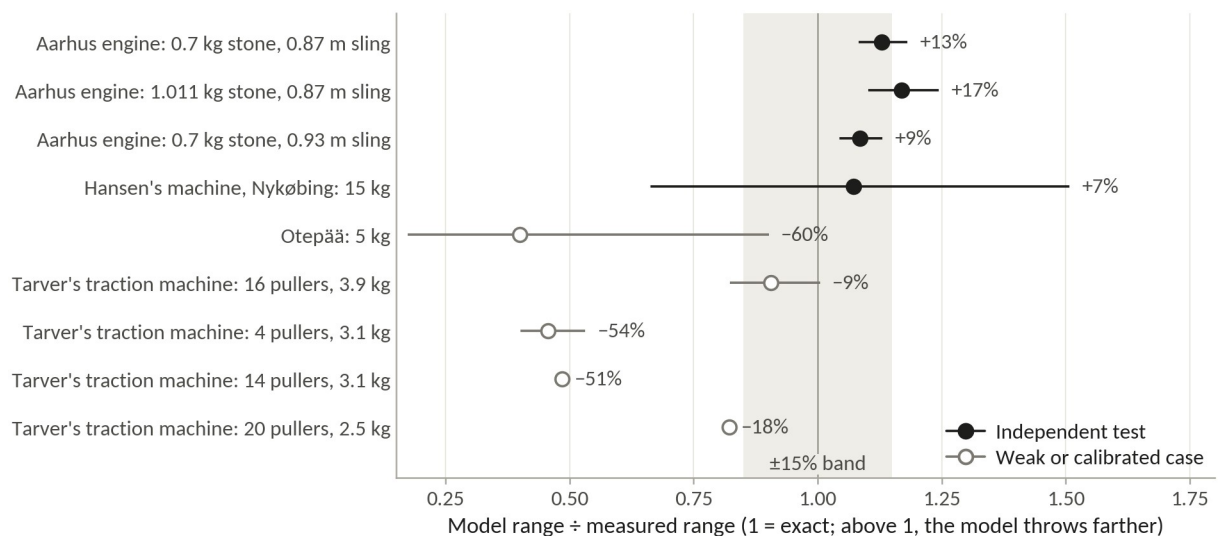


Figure 5. Against real throws the model runs 7 to 17% long where its inputs are known or well estimated, close to the $\pm 15\%$ band, and it fails where they are guesses. Each point is the model's range divided by the measured one, with the 5 to 95% spread over the unknown inputs where these were sampled. Filled points are independent tests,

hollow points weak or calibrated cases. Source: model runs P6-R03, P6-R04, P6-R05 and P6-R11. Image created by Claude [4].

It also follows the builders' rules. A hinge adds 16 to 23% over a bolted weight, propping the weight back adds 8 to 17%, halving the beam's mass adds 16 to 46%, and the best launch angle is 38 to 41°, below the 45° of a vacuum. Where inputs are guesses it can fail, as with Saimre's Otepää machine, which it puts 60% short of its 120 m [49, p. 77]. Our traction-crew model, fitted to one crew, fails for others, so traction ranges here are measured. The simulator in Appendix E runs the same equations and agrees with the model to one part in a hundred million.

6. Designing the Optimal Trebuchet

About 1320 Marino Sanudo wrote that “the side whose artillery hath the longest range will have a vast advantage in action”, as Yule quotes him in his notes to Marco Polo [42, p. 210]. So what should a trebuchet of a given size look like to throw farthest? The Optimal trebuchet answers with any materials at all, the Jerusalem 1187 trebuchet with only what an army at Jerusalem in 1187 could make.

Every design is built at one reference size. Its beam is 10 m long, inside the four to six toises, 8 to 12 m, that Viollet-le-Duc gives for a trebuchet’s beam [47]. No source gives the mass of a stone thrown at Jerusalem in 1187 [80, pp. 159–63], so the stone weighs 20 kg, one of the commonest weights among the stones excavated at Arsuf [58, p. 87]. The counterweight is 150 times the stone, 3,000 kg, between the 133 of Hansen’s Danish machine and the 191 of Horsdal’s large design [56, “The Trials”], [73, table 1]. Each layout is built twice, in period materials and in modern ones.

Every option, from the layout to the timber, was re-optimised on its own terms before it was compared, since a feature that looks poor on someone else’s proportions may be best on its own. A search program tried more than two thousand machines for each question. For every candidate the model also sized the beam to survive its own throw, so no layout could win with a beam too thin to last.

The Tournament

In modern materials the floating arm wins the tournament clearly (Figure 6). Its counterweight drops straight down in guides while its axle rolls forward on a rail, and it throws 885 m against 698 m for the best hinged machine. It needs steel guides and a rail, so in period materials it is out, and two layouts tie: the hinged counterweight at 579 m and a fixed weight on a rolling frame at 586 m.

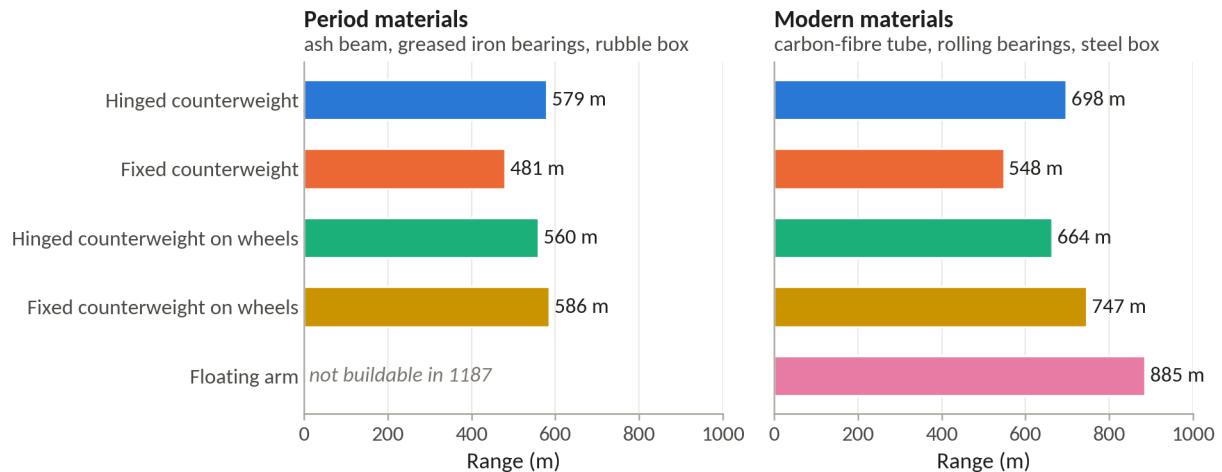


Figure 6. Re-optimised on equal terms, the floating arm wins in modern materials, while in period materials the hinged machine and the fixed weight on a rolling frame tie near 580 m. Each bar is the best machine of that layout with a 10 m beam, a 20 kg stone and 3,000 kg of counterweight, its beam sized to its own loads. Source: model runs P7-R03 and P7-R04. Image created by Claude [4].

Wheels help a fixed weight and hurt a hinged one. A rolling frame lets a fixed weight fall more nearly straight and takes it from 481 to 586 m in period materials. A hinged weight falls straight already, so on wheels it only loses energy to the frame's recoil (560 m against 579 m). The fixed weight's gain also rests on a rolling resistance we had to assume, so the 1187 design stays on a fixed frame.

The Beam Decides

The beam is the biggest lever in the study. A uniform ash beam sized to carry its loads weighs 321 kg, and the machine throws 455 m. The optimiser's beam, deep at the axle where the bending is greatest and tapering to half that depth at the tip, weighs 241 kg and throws 580 m, 27% farther. Every kilogram of beam has to be swung up to speed with the stone, and mass far from the axle costs most. The timber matters far less. Five species fall within 5% of each other. We kept white ash for its toughness and stress it to 40 MPa, under two-fifths of the 106 MPa at which clear ash breaks in bending [81, Ch. 5, table 5-3a]. Carbon fibre wins the modern build, 885 m against 788 m in aluminium.

Box 9. A Dentist's Pumpkin Trebuchet Passes Half a Mile

Steve Seigars, a dentist from Greenfield, New Hampshire, built Yankee Siege with the construction engineer Chuck Willard to bring customers to his farm stand [82, Paras. 6, 11], [83]. It stood 15 to 17 m (50 to 55 ft) tall on 3 m (10 ft) iron wheels and was designed for loads of up to 136 kg (300 lb), yet it made its name with pumpkins [82, Paras. 9–10], [83]. At the 2004 World Championship Punkin Chunkin in Delaware [82, Para. 2], [84], it threw one 425 m (1,394 ft).

Then Seigars went after the arm. In 2008, after fitting a lighter one, the machine threw 578 m [85]. By 2009 he had cut the arm to 272 kg (600 lb) and moved its mass towards the axle to make it easier to swing [70], which is what our optimiser does when it tapers the ash beam. Its successor, Yankee Siege II, set a trebuchet record of 864 m (2,835.81 ft), according to the American Institute of Aeronautics and Astronautics [86], and in 2016 the Associated Press reported Seigars sending pumpkins “more than a half mile” [87]. A pumpkin is not a stone, though, and none of these distances can be set beside a medieval engine's.

No magic ratio comes out of the search. The best period machine sits where four limits meet: the hanger as long as the ground allows, the frame as tall as we allow, the hinge pin at the 5 tonne load we set for a timber box on iron straps, and the beam at its design stress. A taller frame would always help, since a higher axle lets the weight fall farther, and lowering this one from 7.5 to 5.9 m costs 25%. We capped the axle at three-quarters of the beam's length because tall uprights are hard to build and brace, not because the physics asks for it. The result has an arm ratio of 4.8, a hanger 2.8 times the short arm, a sling as long as the long arm and a start 26° from the vertical. All but the sling fall inside the ranges of Horsdal's independently optimised designs, and the sling is only a little longer than his 0.86 to 0.95 times the long arm [46], [74, table 4].

The counterweight ratio of 150 is a judgement too. Range keeps rising with the weight, though slowly: at 150 each extra 1% of counterweight buys about 0.5% more range, and at 400 still 0.37%. With no point where the gains stop, 150 rests on precedent and logistics, since 3 tonnes of rubble fills a box 1.1 m on a side that a capstan can haul.

Bearings are worth a few per cent: greased iron journals give 580 m, against 553 m for dry wood and 594 m for rolling bearings, which on this geometry push the hinge load just past its limit. Propping the counterweight back 15° adds 2%, and the trough adds 9% over a stone that starts hanging in its sling. A thousand machines built with random errors of up to 5% in

every length and mass lose a median 1.0% of range for the 1187 design and 0.6% for the Optimal.

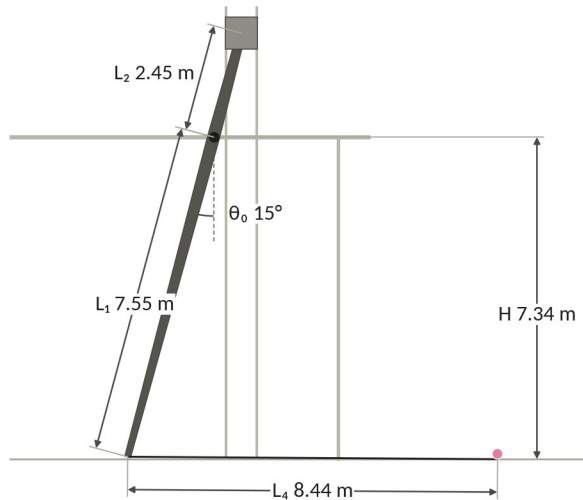
Table 3 and Figure 7 set the two designs side by side. The Optimal trebuchet, a floating arm with a carbon-fibre beam of 99 kg, throws the 20 kg stone 885 m, an R^* of 88.5, at an efficiency of 0.91. The Jerusalem 1187 trebuchet hangs its counterweight from a tapered ash beam on an oak frame and throws 580 m, R^* 58.0, at 0.75. With the model about 10 to 15% optimistic at laboratory scale, that 580 m is better read as about 500 m. The floating arm and the rolling frames have no measured case to check them against, and the limits on hinge load and frame height are our own, so those results deserve less weight than the rest.

Table 3. The two designs at a glance, at the reference scale. Ranges are model values, about 10 to 15% optimistic. Source: the design specifications, model run P7-R12.

	Optimal trebuchet	Jerusalem 1187 trebuchet
Layout	Floating arm: weight falls in guides, axle rolls on a rail	Hinged counterweight on a fixed frame
Arm ratio L_1/L_2	3.1	4.8
Hanger ÷ short arm L_3/L_2	none	2.8
Sling ÷ long arm L_4/L_1	1.1	1.0
Start angle θ_0 (from the vertical)	15°	26°
Long arm L_1 , short arm L_2 (m)	7.55, 2.45	8.28, 1.72
Hanger L_3 , sling L_4 (m)	none, 8.44	4.86, 8.34
Axle height (m)	7.34	7.46
Counterweight, stone (kg)	3,000, 20	3,000, 20
Beam	Carbon-fibre tube, 28 cm at the axle, tapered, 99 kg	White ash, 27 × 19 cm at the axle, tapered, 241 kg
Counterweight box	Steel, 0.72 m cube	Stone rubble, 1.14 m cube
Frame	Steel legs	Oak legs, 17 cm square
Axle bearings	Rolling bearings, 14 cm steel axle	Greased hardwood, 15 cm wrought-iron journals
Sling	Two legs of 8 mm HMPE rope	Two legs of 25 mm hemp rope
Range (m)	885	580
R^*	88.5	58.0
Efficiency η	0.91	0.75
Release speed and angle	114 m/s at 38°	85 m/s at 40°

Optimal trebuchet

floating arm, modern materials
carbon-fibre composite beam, 99 kg; range 885 m



Jerusalem 1187 trebuchet

hinged counterweight, period materials
white ash beam, 241 kg; range 580 m

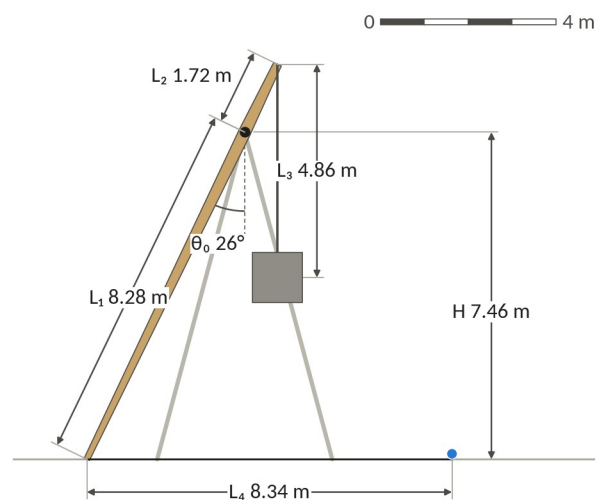


Figure 7. The Optimal and Jerusalem 1187 trebuchets drawn to the same scale and cocked for the throw. Both have a 10 m beam and an axle about 7.4 m up, and they differ in how the counterweight falls and in what the beam is made of. Source: model run P7-R12. Image created by Claude [4].

A siege engineer wants more than distance. Figure 8 trades stone mass against range, with the sling and prong re-tuned for each stone. The 1187 machine throws a 4 kg stone 791 m and a 40 kg stone 340 m. Its 20 kg stone strikes with 47 kJ and the 40 kg stone with 65 kJ, but past about 40 kg the impact energy barely grows, because the machine can no longer bring a heavier stone up to speed. Chapter 8 sets these numbers against the wall Saladin attacked.

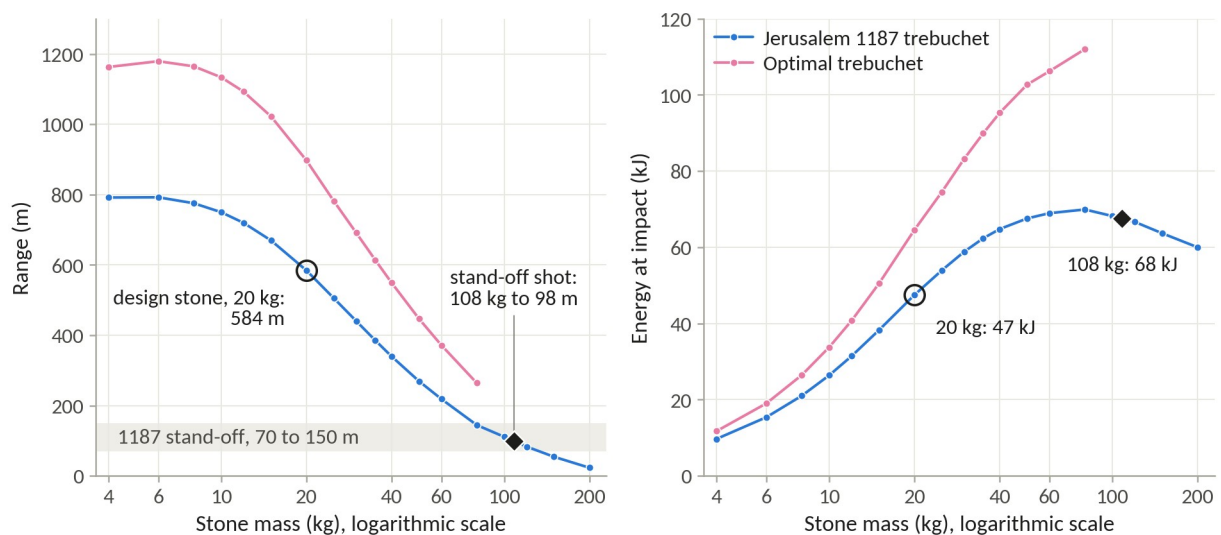


Figure 8. Heavier stones fly shorter, and the energy they strike with stops growing past about 40 kg. Range (left) and impact energy (right) against stone mass for both designs, with the sling and prong re-tuned for each stone,

which gives the 20 kg design stone 584 m here against the design's 580 m. The 108 kg stone that lands within the 1187 stand-off band is marked. Source: model runs P7-R11 and P8-R05. Image created by Claude [4].

7. History against the Optimum

Saimre measured Hansen's Danish trebuchet at Nykøbing from its published blueprint and found its axle "only 3.15 m" off the ground. Raising it by one metre, he calculated, "would result in an increase of the theoretical maximum range by 17%" [49, p. 75]. Our model gives 18% for the same change. Two independent calculations point to the frame, not the proportions.

Five machines are documented well enough to run, and three agree with the model as built, given its known optimism of 10 to 15%: the Aarhus laboratory engine (13% long), Hansen's machine at Nykøbing (7% long) and the Visby reconstruction (4% short of the range its builders report) [88]. Varbola's claimed 10 kg stone to 120 m [49, pp. 75–76] sits at the low edge of the model's envelope of 91 to 251 m and would need a heavier beam or a poorly tuned release. Black Widow's single shot of 270 m [89, p. 4] lies beyond every machine we sampled for it. Favé's Vincennes engine, printed with a 30 cm short arm [49, p. 74], could not have started a beam over about 250 to 390 kg and is set aside. Only the Aarhus beam was weighed, and Black Widow's was estimated by its builder. For the other three we assume a beam weighing a fifth of the counterweight, and every model number for them carries that assumption.

As reported, the medieval-pattern reconstructions scored an R^* of 16 to 24 for their size, 28 to 41% of the 1187 design's 58.0 (Figure 9). Scaled to the reference machine, which removes the accidents of size and ballast, the three reach R^* 19 to 26 with their own beams by our fifth-of-the-counterweight rule. Given an ash beam sized to its loads, as the designs have, they reach 36 to 45. Black Widow goes from 30 to 50 and the Aarhus engine from 42 to 57, almost the 1187 design itself.

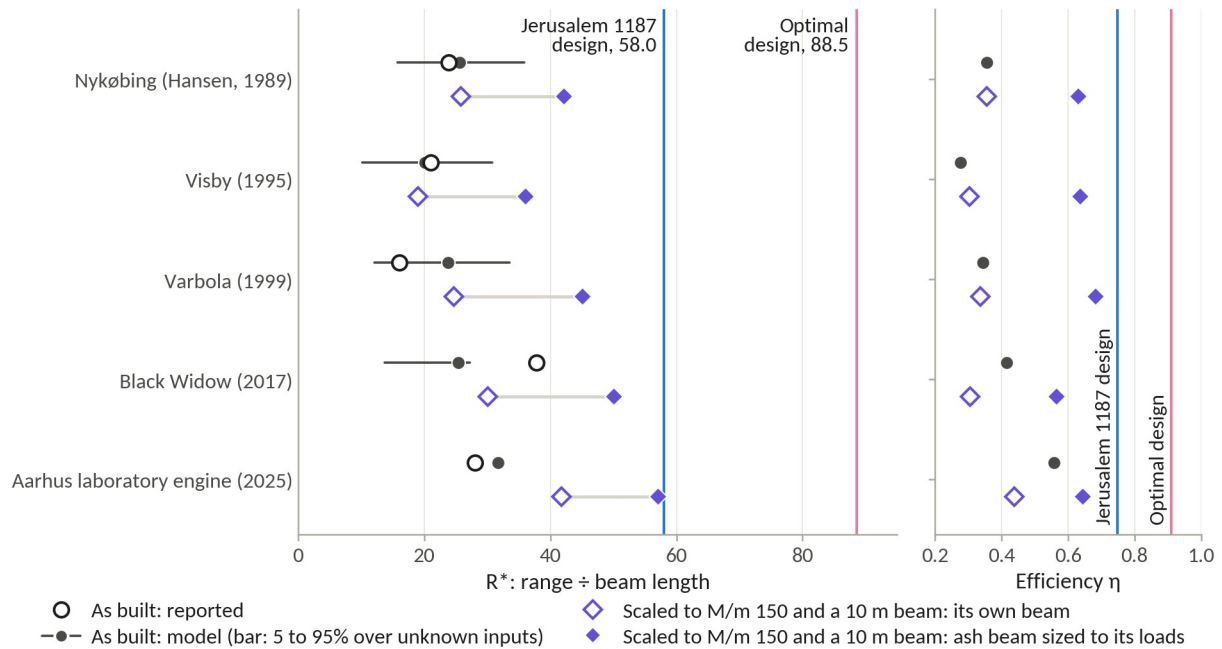


Figure 9. Reconstructions reached 28 to 41% of the 1187 design's score for their size, and a beam sized to its loads closes most of the gap. R^* (left) and efficiency η (right) for five machines: as reported, as modelled at their own size, and scaled to the reference machine with their own beams and with an ash beam sized to its loads. Where no beam was weighed (Nykøbing, Visby, Varbola) we assume one a fifth of the counterweight's mass. Black Widow's beam is its builder's estimate and the Aarhus beam was measured. Source: model runs P8-R01 and P8-R02, designs from P7-R12. Image created by Claude [4].

Where the Gap Goes

Figure 10 walks Hansen's machine, at a counterweight ratio of 150 and R^* 25.8, to the optimum one change at a time. An ash beam sized to its loads adds 16.4 and raising the frame to the design's height adds 14.3, while re-proportioning the machine for its own frame adds only 1.5. The beam supplies 51% of the gap and the frame height 44%, and across all five machines the beam supplies 44 to 94% and the frame height 15 to 44%. This is the wrong frame of the paper's title, meaning the machine's whole structure: a beam too heavy for its work, carried on uprights too short to let the weight fall far. The beam's share is only as firm as our guess, and at half the assumed weight its step shrinks from 16.4 to 4.5.

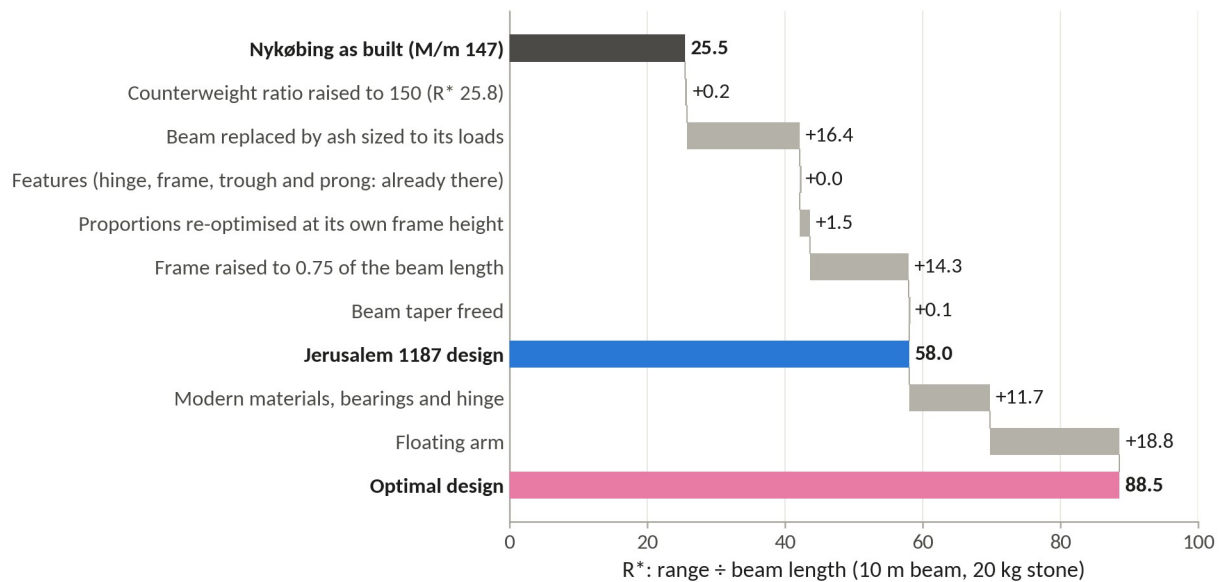


Figure 10. From the Danish machine to the optimum, step by step: the beam and the frame height do most of the work, and better proportions add little. R* at the reference scale, starting from Nykøbing as built with a beam a fifth of its counterweight (our assumption) and ending at the Optimal trebuchet. Source: model run P8-R04, with the last two steps from P7-R04 and P7-R12. Image created by Claude [4].

The proportions were not the problem. Held to Hansen's frame height, an axle at 0.45 of the beam's length, the best machine our optimiser finds has an arm ratio of 4.7, a hanger 1.0 times the short arm, a sling 0.76 times the long arm and a start 57° from the vertical. Hansen's has 4.9, 1.1, 0.85 and 58° [49, p. 75], [78, p. 3], and with the same ash beam it throws within 4% of that best machine (Figure 11). The short hanger and steep start that look wrong beside the 1187 design are right for a low frame. The low frame itself costs a quarter of the range: held to 0.45 of the beam instead of 0.75, the best period machine loses 24%.

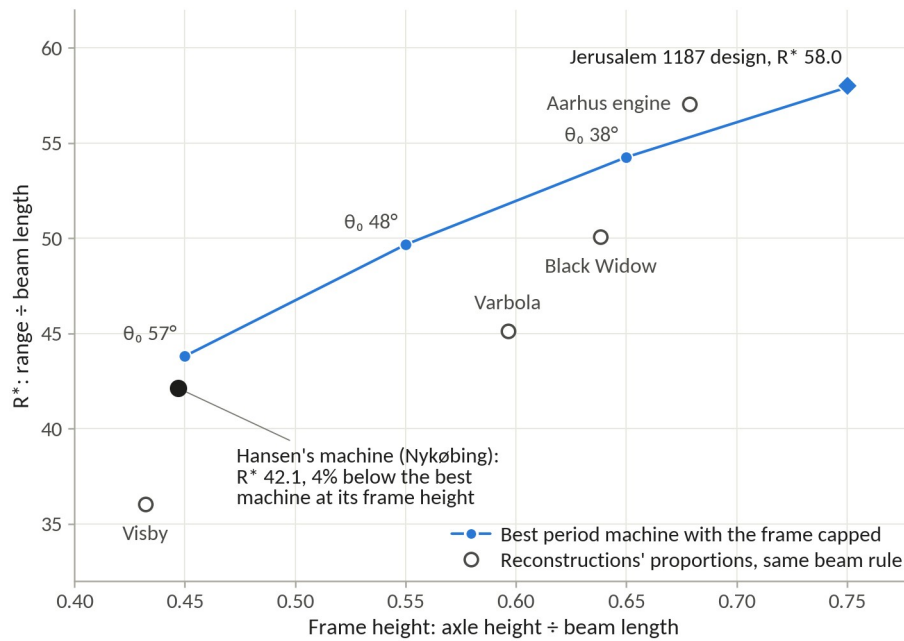


Figure 11. At its own frame height the Danish machine's proportions are within 4% of the best possible, and a low frame costs about a quarter of the range. The line is the best period machine when the axle may rise only to the given fraction of the beam's length, with its start angle marked. The circles are the reconstructions' proportions with the same ash-beam rule. Source: model runs P8-R03 and P8-R02. Image created by Claude [4].

Nor were features missing. Hansen's machine already had a hinged counterweight and a prong release, and on its proportions a fixed weight would cost 14% and a prong set 10° early 5%. Dry wooden bearings would cost 3%.

Why, then, did builders keep their frames low and their beams heavy? The sources do not say, and what follows is our inference. A tall frame needs long timbers and stands less steadily than a squat one. A heavy beam rarely breaks, and a broken beam ends an engine's work. Favé's frame failed after four shots [49, p. 74], while Hansen's fired about 120 shots and "functioned nearly flawlessly in all of them" [56, "The Trials"].

Built for Weight

The larger reason, we think, is purpose. Builders wanted heavy shot, and a heavy stone flies short. At Shawbak two limestone counterweight blocks weigh about 450 kg and 62 kg, and the stone missiles found there average about 60 kg [59, pp. 27–31]. The weighed stones from Arsuf run mostly from 8 to 48 kg [58, p. 87], and eight shots excavated outside Kenilworth Castle weigh from 1 kg to 105 kg [57], though Kenilworth fell in 1266 to disease and starvation, not to

its engines [90]. Giles of Rome's fixed-weight engine shot more regularly [63, p. 29], as Chapter 4 noted, which would have suited a crew aiming at one stretch of wall.

Fulton argues that traction engines served against people [26] and that even the largest trebuchets were not effective breaching engines by the time the Franks were pushed out of the Holy Land [53, vii]. Wherever a fortress was taken in his period, it fell by other means, most often mining [26]. Purton finds it "questionable" that carefully carved stones of more than 100 kg had no serious physical effect, and notes that Fulton himself wavers [26]. At Toulouse on 25 June 1218 a stone from the defenders' engine killed Simon de Montfort [91], and Peter of les Vaux-de-Cernay calls that engine a mangonel [92]. Box 10 shows engines and miners working together at Acre in 1191.

Box 10. Bad Neighbour, Bad Kinsman and the Engine of God

At the siege of Acre in 1191 the stone-throwers had names. The *Itinerary of King Richard*, once attributed to Geoffrey de Vinsauf, reports that the French king built a petraria "of superior quality" called Bad Neighbour [93, p. 205]. The Turkish defenders answered with one they called Bad Kinsman, which "by its violent casts, often broke 'Bad Neighbour' in pieces" [93, p. 205]. The king rebuilt it each time, until it broke down part of the main wall and shook the tower called Maledictum [93, p. 205].

Another engine, built at common expense, was known as the petraria of God. A priest preached beside it and collected money to repair it and to hire men to bring stones, and it was credited with shaking down about two poles' length of the tower's wall, roughly 10 m if the pole is the English one [93, p. 205]. The engines did not work alone. Richard's sappers carried a mine under a tower at which a petraria was aimed and fired the props, and with further blows from the engine the tower fell to the ground [93, p. 210]. The chronicle does not say whether these engines were pulled by crews or driven by counterweights.

8. Jerusalem, 1187

On Friday 25 September 1187, 20 Rajab 583 in the Arabic sources, Saladin's mangonels went up against the north-east angle of Jerusalem, installed in the night and working by morning, as Ibn al-Athīr tells it [94, "Salah al-Din's Attack"]. A Latin eyewitness saw olive branches "piled between the city and the engines", and behind them "the Turks unceasingly hurled rocks forcefully against the ramparts" [80, pp. 159–63].

Hattin had destroyed the kingdom's field army on 4 July, and Balian of Ibelin now held Jerusalem with so few knights that he knighted fifty sons of the nobility [94, "Jerusalem Between July and September 1187"]. The siege opened on 20 September on the west, opposite the Tower of David [80, pp. 159–63]. After five days (a week, says Ernoul), Saladin judged that he could do no damage there and moved round to the north-east [80, pp. 159–63], [94, "Salah al-Din's Attack"].

In the Latin account Saladin had engines built behind the branch screens, and that evening he sent engineers with iron tools to the foot of the walls. At daybreak, while the engines threw, his men began to break down the corner of a tower [80, pp. 159–63]. In the Arabic accounts the mangonels destroyed the tops of the towers and drove the defenders from their posts, and in two days the sappers opened a breach about 30 m long [94, "Salah al-Din's Attack"]. It was where the First Crusaders had entered in 1099 (Chapter 2), and the great cross set up there fell with the wall [94, "Salah al-Din's Attack"]. Inside, no man would keep watch for a night, even for a hundred besants [80, pp. 159–63].

Balian won terms by threatening to destroy the Dome of the Rock and kill the Muslim prisoners in the city [94, "The Surrender of Jerusalem"]. Each man would pay ten dinars and each woman five, within forty days [80, pp. 159–63], [94, "The Surrender of Jerusalem"]. Jerusalem surrendered on Friday 2 October, twelve days after the siege began [80, pp. 159–63], [94, "The Surrender of Jerusalem"]. It was the last siege of the walled city (Box 11).

Every account credits the breach to the sappers, and none gives the size of an engine or the weight of a stone [80, pp. 159–63], [94, "Salah al-Din's Attack"]. To ask what artillery could have done at this wall, we have to bring our own.

Box 11. The Sultan Who Pulled Down His Own Walls

After 1187 the Ayyubids gave the city new walls with towers and a moat, work carried on after Saladin by his brother al-ʿĀdil and his nephew al-Muʿaẓẓam ʿĪsā [95, p. 446]. They did not stand for long. During the Fifth Crusade, fearing that the crusaders would take the city, al-Muʿaẓẓam had the whole circuit pulled down, sparing only the Tower of David, the citadel [96, p. 67]. The work was done in 1219 and again in 1220, and excavators have found the fallen masonry at the foot of the forewall and near Herod's Gate [95, p. 447]. When Frederick II recovered Jerusalem by treaty in 1229, the walls were not rebuilt [97].

The fighting went on without them. In December 1239 an-Nāṣir Dāwūd's army walked into an unwallled city, took the fortress the crusaders had built after a 21-day siege and starved out the Tower of David, then brought in miners to demolish what was left [96, p. 71]. In July 1244 the Khwarazmians entered the city, and the citadel held out until 23 August [96, p. 81]. Both were sieges of a citadel in an open city. Jerusalem stayed unwallled for about 300 years, until Süleyman built the wall that stands today [95, p. 447], [98]. Saladin's siege remains the last siege of the walled city.

The Wall and the Stand-Off

On the north the sappers had to cross a rock-hewn moat 14 to 20 m wide and more than 9 m deep, then pass a forewall 3.2 to 4.5 m thick standing 8 to 10 m in front of the main wall [95, pp. 424, 425, 436, 443, 445]. The main wall is unmeasured, so our working values of 11 m high (9 to 15 m) and 2.5 m thick (2.0 to 3.2 m) lean on walls of its age such as Pisa's, which have a weak core and average 11 m by 2.2 m [99, Sec. 2]. We take it to be faced in meleke, the Old City's main building stone [100, Sec. 1.1], which is light ($1,976 \text{ kg/m}^3$) and weak, crushing at 11 to 16 MPa [101, p. 15].

No source says how far back the engines stood. Saladin's tent was within reach of the defenders' weapons and his engines worked behind screens [80, pp. 159–63], [94, "Salah al-Din's Attack"]. Shooting a later steel siege crossbow, Ralph Payne-Gallwey reached 421 m (460 yards) [102, Ch. III, 14–15]. Saladin's engines were most probably traction machines, since the counterweight was only then appearing [26], and these threw "100 meters or more" [21, p. 68]. We take the stand-off, the distance from engine to wall, as 70 to 150 m, a judgement rather than a measurement.

The 1187 Design at the Wall

Our 1187 design from Chapter 6, a hinged counterweight machine with a tall oak frame and a tapered 10 m ash beam, throws a 20 kg stone 580 m in our model. The model runs about 10 to 15% long, so about 500 m is the safer figure. Either way it out-ranges any crossbow, and from 421 m it could still throw 32 kg (Table 4 and Figure 12).

From out of range, though, it would mostly miss. A stone coming down at 48° drops past the wall's 11 m height while travelling only about 10 m forward, and small differences in the stones and in the moment of release shift the landing point by a typical 13 to 16 m. Only 27 to 30% of these shots strike the face. About 40% fly over into the city, still dangerous to people and roofs, and the rest fall short.

Table 4. What the 1187 design could have done at Jerusalem's wall: from beyond the crossbows most shots miss the 11 m face, and from the stand-off most hit. Ranges are model values, about 10 to 15% optimistic. Shares are from 200 simulated shots per setting aimed 8 m up the face, without wind. Source: model runs P7-R12, P8-R05 and P8-R06.

Stone (kg)	Firing position	Range (m)	Impact energy (kJ)	Shots striking the face
20	Full power, out of range	580	47	27%
32	Just beyond the steel crossbow	421	60	29.5%
108	Stand-off, heaviest stone	98	68	68%
20	Stand-off, late release (flat shot)	100	71	99.5%

Moved in to the stand-off, the machine trades range for weight. With its sling and release re-tuned it throws 108 kg to 98 m or 75 kg to 165 m, striking with 68 to 69 kJ, and 68% of the 108 kg stones hit the face. Released late, a 20 kg stone flies almost flat and hits the face 99.5% of the time, with 71 kJ. The sling, not the beam, is the weak point. It pulls 28% harder at 108 kg and 64% harder in the flat shot, so a crew would have wanted thicker rope.

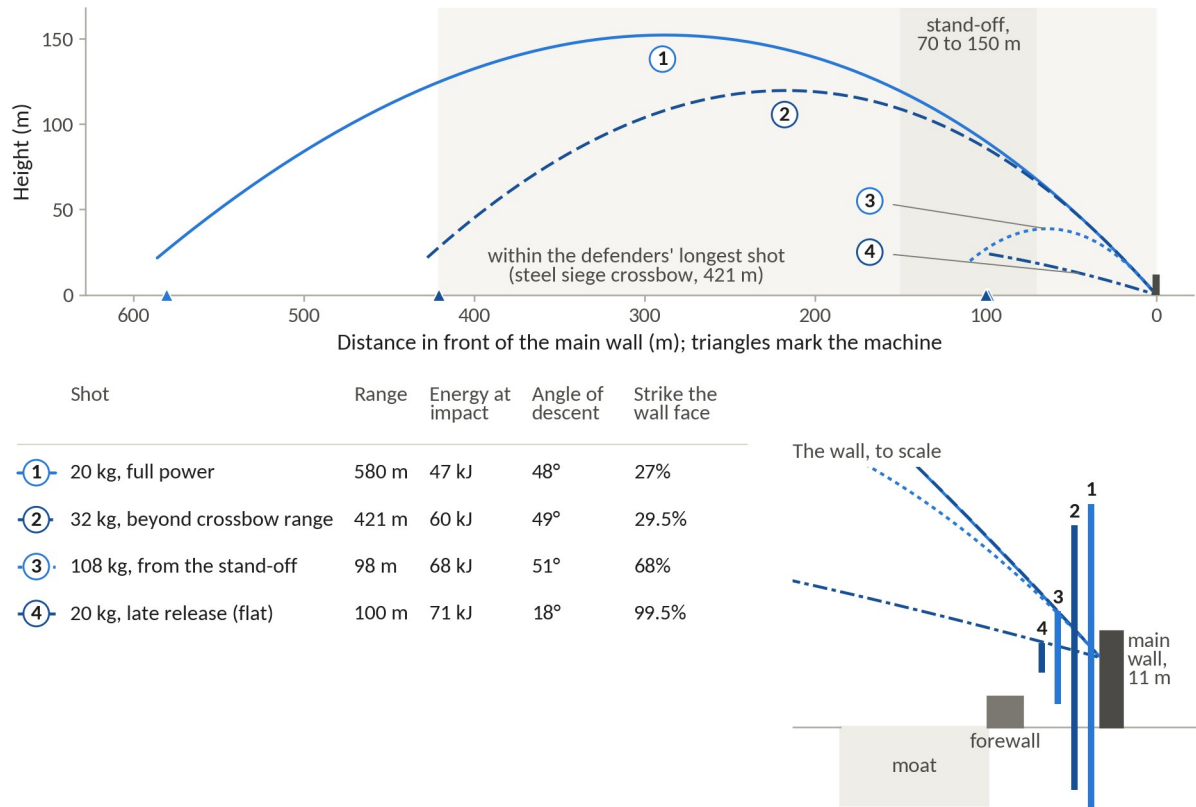


Figure 12. Out of crossbow range the 1187 design mostly misses the wall, and from the stand-off it mostly hits. Above, four flight paths to scale for the settings of Table 4, with the 70 to 150 m stand-off and the 421 m crossbow shot marked. The angle of descent is how steeply each stone comes down. Inset, the wall and its outer defences to scale, with each setting's shot-to-shot spread (one standard deviation) as a bar. Source: model runs P8-R05 and P8-R06 (200 shots per setting, no wind, wall from the working values). Image created by Claude [4].

In energy the design matches Josephus's engines. Its 20 kg stone strikes with 47 kJ. The Prologue's talent (26 kg) thrown two stades (370 m) at its best angle, allowing for air drag, needs at least 55 kJ at launch and strikes with about 41 kJ [5, 5.270]. A typical 5 kg stone from the Third Wall, thrown an assumed 300 m, strikes with about 6 kJ, an eighth of the design's shot [7, p. 110].

Traction engines fall further short. Al-Ṭarsūsī, writing about 1187, gives his engine about 120 m at most [48, p. 149], and a traction reconstruction threw 3.1 kg to 145 m and 4.7 kg to about 78 m [48, p. 162], each throw arriving with about 2 kJ. The 1187 design throws a stone four to seven times heavier, four to seven times as far. At the top of Figure 13, a 20 cm iron cannon ball striking at 200 m/s carries 660 kJ [99, Sec. 4], fourteen times the design's 20 kg shot.

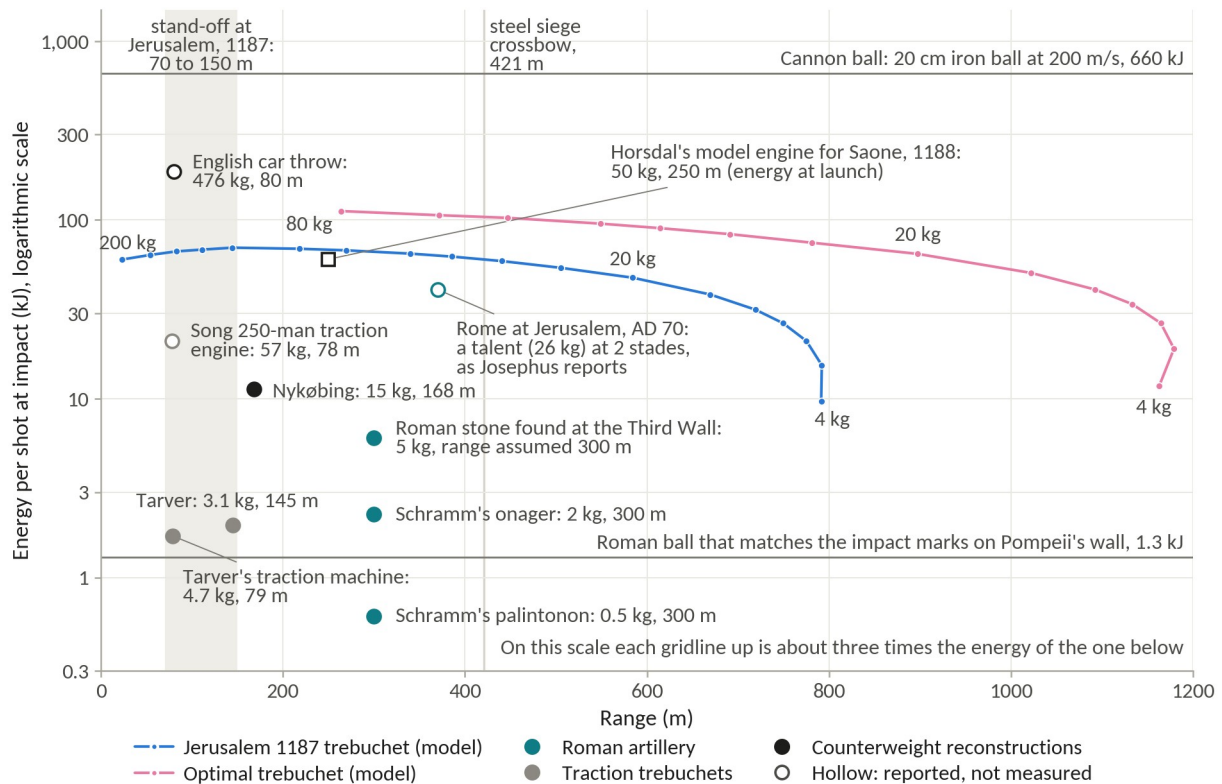


Figure 13. A 20 kg shot from the 1187 design strikes with about the energy that Josephus's talent at two stades implies, more than twenty times a measured traction throw and a fourteenth of a cannon ball. Energy at impact is plotted against range on a logarithmic scale. The curves follow each design as its stone gets heavier (labels give the stone mass), and each point is the least energy that reaches the reported or measured range, hollow where the throw is reported rather than measured. Horsdal's engine for Saone (1188), like ours, is a modern optimum and not a reconstruction. Source: model runs P7-R11, P8-R05 and P8-R07. Image created by Claude [4].

Could It Have Broken the Wall?

Probably not on its own, and not in the twelve days. It would have wrecked the tower tops from beyond the crossbows and covered the sappers, doing the mangonels' work faster and from safety. Our confidence is Low to Medium.

Each hit still carries only a tenth to a fourteenth of a cannon ball's energy, and it was gunpowder that forced builders to thicken their walls [99, abstract]. A soft inner core like Pisa's also makes impact damage shallower but wider in Tateo and Casolo's model [99, abstract], so a clean hole is harder to open. The history agrees. Fortresses of the crusade era fell most often to mining [26]. At Hansen's pace of one shot every ten minutes [56], a twelve-hour day at the stand-off puts 50 to 70 stones on the face, and nothing suggests that would match 30 m of sapped wall in two days.

The verdict leans on a model that runs 10 to 15% long, a borrowed wall, a judged stand-off and no tested link between impact energy and a breach. What holds it up is that the chroniclers' sappers and the comparison with gunpowder agree.

Epilogue: Gunpowder

On the night of 26 to 27 September 1687 a Venetian army was bombarding the Ottoman garrison on the Acropolis of Athens, which had moved its gunpowder into the Parthenon [103, p. 17]. One bomb passed through an opening in the roof and set the powder off, killing about 300 men [103, p. 17]. Six columns fell on the south side and eight on the north, and of the east colonnade only one column was left. Three-fifths of the frieze came down [103, p. 17]. Morosini called it “a fortunate shot” [104].

Guns had long since replaced the trebuchet, in a change that took nearly two centuries [21, p. 71]. In 1453 Orban’s great gun, firing a granite ball of about 600 kg in a modern reconstruction, helped the Ottomans breach Constantinople’s land wall [105, pp. 481–496], [106]. Builders answered with thicker walls [99, abstract], and when Jerusalem was walled again, under Süleyman in 1538 to 1540, it was in the age of guns [98]. At Tenochtitlán in 1521, with ammunition low, the Spaniards built a trebuchet. Its first stone went straight up and wrecked the machine in Saimre’s account, or fell behind the marketplace at Xomolco in the Nahua one [49, p. 65], [107, 12.38]. Gibraltar in 1779 is the last reported use [49, p. 65], [56].

The best trebuchet that physics and modern materials allow is a floating arm, which in our model throws a 20 kg stone 885 m from a 10 m beam. In the materials of 1187 the best is a tall hinged machine that throws the same stone 580 m, about 500 m allowing for the model’s optimism, and strikes with about the energy Josephus’s talent implies, a fourteenth of a cannon ball’s. The builders of history got the proportions nearly right for frames that were too low, and they lacked a light, strong beam. Mostly they wanted heavy stones more than distance. At Jerusalem the best machine they could have built would have ruled the top of the wall without breaking it. The sappers would still have made the breach.

Building either design, from CAD model to full-size trial, was beyond this project. Hansen’s printed report may hold the shot-by-shot data our checks lacked. Jerusalem’s twelfth-century wall still needs measuring, perhaps from excavation reports we could not open. Meanwhile the simulator in Appendix E lets any reader fire both designs at Saladin’s wall.

Appendix A. Equations, Notation and Glossary

The model behind Chapters 5 to 8 rests on one set of equations of motion, derived by computer algebra and checked against 21 limiting cases before any machine was thrown. This appendix shows their structure and defines the scores the paper quotes. Table A1 lists the symbols and Table A2 the names of the engines.

Notation and Equations of Motion

All quantities are in SI units, with angles in degrees. Every angle is measured from the downward vertical and is positive in the throwing sense, clockwise when the target is on the right of the page. A link at angle θ points along

so 0° hangs straight down, 90° points away from the target and 180° points straight up. The Jerusalem 1187 design is cocked at $\theta_0 = 26^\circ$, its long arm pointing down and back.

Sources differ. Horsdal's paper on internal forces measures angles from the horizontal, so his reference machine starts at -50° where ours starts at 40° [73, table 1]. Saimre's 44° to the horizon at Otepää is a θ_0 of 46° here [49, p. 76].

Forces are quoted as multiples of a weight. Mg is the counterweight's weight (M times g , not megagrams) and mg the stone's, so a hinge load of $5 Mg$ is five times the counterweight's weight. In these units, geometrically similar machines carry the same loads at every size.

Table A1. Symbols used in the paper and the model. Angles are absolute, measured from the downward vertical. In prose subscripts are set as L_1 , in equations as $\cdot$.

Symbol	Meaning	Unit
L_1	Long arm: axle to the sling's attachment at the tip	m
L_2	Short arm: axle to the counterweight's hinge (fixed weight: to its centre of mass; floating arm: to its pin; traction: to the ropes)	m
L_3	Hanger: hinge to the counterweight's centre of mass, 0 for a fixed counterweight	m
L_4	Sling: tip to the stone's centre, sling taut	m
L_b	Beam length, $L_1 + L_2$	m
L_{CM}	Axle to the beam's centre of mass, along the long arm	m
H	Axle height at the start (floating arm: height of its track)	m
y_t	Trough height above the ground, 0 by default	m
d	Stone diameter	m
l, λ_3, λ_4	Arm, hanger and sling ratios: L_1/L_2 , L_3/L_2 and L_4/L_1	none

Symbol	Meaning	Unit
M	Counterweight mass	kg
m	Stone mass	kg
M/m	Counterweight ratio	none
m_b	Beam mass	kg
I_b	Beam's moment of inertia about the axle	$\text{kg}\cdot\text{m}^2$
I_c	Counterweight's moment of inertia about its own centre of mass	$\text{kg}\cdot\text{m}^2$
m_f	Effective mass of a rolling frame, wheels included	kg
ρ_s, ρ_f	Density of the stone and bulk density of the counterweight's fill	kg/m^3
θ, θ_0	Long-arm angle, and its value when cocked	°
ψ, ψ_0	Hanger angle (0 when hanging straight down), and its starting value	°
φ	Sling angle (-90° when lying in the trough towards the target)	°
δ	Sling-to-arm angle, $\varphi - \theta$	°
β	Prong angle, between the arm's axis and the release prong	°
δ_r	Sling-to-arm angle at which the ring leaves the prong	°
x	Displacement of a rolling frame	m
q	The coordinates of a layout (θ, ψ, φ and x, as present)	° or m
t_{lo}, t_r	Lift-off and release times	s
g	Acceleration due to gravity, 9.81	m/s^2
ρ_a	Air density	kg/m^3
C_d	Stone's drag coefficient, on the area $\pi d^2/4$	none
Re	Reynolds number, $v d/\nu$, with ν the kinematic viscosity of air	none
α_s	Area factor for the drag of sling and pouch inside the machine	none
$C_{d,b}$	Beam's drag coefficient (optional)	none
μ_a, r_a	Axle friction coefficient and journal radius	none, m
μ_h, r_h	Hinge friction coefficient and pin radius	none, m
μ_t	Friction between stone and trough	none
μ_p	Friction between the sling's ring and the prong	none
C_{rr}	Rolling resistance of wheels or rollers	none
μ_g	Friction in the floating arm's guides	none
F_R	Axle reaction, the frame's force on the beam	N
F_{perp}	Component of F_R across the beam, the bending load at the axle	N
F_H	Hinge force between short arm and hanger	N
T_s	Sling tension	N
λ	The trough's upward push on the stone	N
N_p, F_p, α_p	Traction: number of pullers, their total rope force, and the rope's angle from the vertical	none, N, °
R	Range on level ground, from the point below the axle at the start	m
R^*	Size-free range, $R/(L_1 + L_2)$	none
$R^*_{\text{ideal}}, R^*_{\text{ref}}$	R^* in vacuum without friction, and R^* with drag, friction and a sized beam at the reference scale	none
v_r, α_r, h_r	Stone's speed, climb angle and height at release	m/s , °, m
E_i	Impact energy, $\frac{1}{2} m v^2$ at impact	J

Symbol	Meaning	Unit
Δh_{CW}	The counterweight's available drop	m
η	Efficiency: the stone's kinetic energy at release over $M g \Delta h_{\text{CW}}$	none
η_{H}	Horsdal's efficiency	none
η_{R}	Range efficiency (Siano, Saimre)	none

Each machine is a few rigid bodies joined by pins: a beam of mass m_b turning on an axle, a counterweight of mass M , a sling that is massless and cannot stretch while taut, and a stone treated as a point of mass m . The layouts differ in the coordinates q that describe them:

- fixed counterweight: θ and φ
- hinged counterweight: θ , ψ and φ
- either of these on free-rolling wheels: add the frame's displacement x
- floating arm: θ and φ , with the counterweight in vertical guides and the axle rolling on a track
- traction: θ and φ , with the crew's rope force in place of a counterweight

Positions follow from the geometry. With the axle at $P = (x, H)$, and $x = 0$ on a fixed frame, the tip S , the short-arm end K , the counterweight C and the stone p lie at

Lagrange's method turns energies into equations of motion. For the hinged counterweight on a fixed frame the kinetic energy T and the potential energy V are

With , each coordinate obeys

where Q_i collects the losses and any crew force. Together the equations take the form

in which the bold is the mass matrix, not the counterweight. What drives the throw is the moment in the θ equation, the counterweight's turning moment less those of the beam and the stone.

A fixed counterweight is the hinged one with $L_3 = 0$ and $\psi = \theta$. On wheels, x enters only through speeds, so without rolling resistance the machine's horizontal momentum is

conserved. The frame rolls to keep the centre of mass from moving sideways, and the counterweight falls more nearly straight. In the floating arm the pin fixes the axle at $P = (-L_2 \sin \theta, H)$, so the counterweight drops straight down.

The algebra was done in SymPy, so no equation was typed by hand, and the same generated code drives the Python model and the simulator. For a hinged machine at rest in the trough, the arm's starting acceleration matches Horsdal's closed form, rewritten for our angles [73, eq. 19]:

On his 19.1 t design the model gives 5.78 rad/s^2 against his 5.79 rad/s^2 [73, Sec. 8.5], and without losses it conserves energy to one part in 10^{10} (run P5-R01).

A Throw in Four Stages

A throw is integrated in four stages, shown for one machine in Figure 3 (Chapter 5) and for every layout in Figure A1 below.

In the trough stage the stone starts at rest, with the sling straight and pointing at the target. The constraint holds the stone at trough height and adds the trough's upward push λ to the equations:

The accelerations and λ are solved together, with a small correction that keeps the stone on the trough to within 10^{-11} m . The stone lifts off when λ reaches zero.

In the swing stage the stone hangs in the sling, and the model checks that the sling stays taut ($T_s > 0$) and that neither stone nor counterweight strikes the ground or the frame.

Release is set by a prong at the tip of the arm. A ring at the free end of the sling sits on the prong, and the sling pulls the ring along its own line. The ring stays on while the sling makes more than a right angle with the prong and slides off once that angle falls below $90^\circ - \arctan \mu_p$. With the prong at an angle β to the arm, the stone is released the first time

A larger β releases later. The model can instead find the ideal release, the instant that gives the longest range, which bounds every prong setting and stands for a crew that has tuned its prong over many shots.

In flight the stone is a point mass with quadratic drag in still air,

until it returns to the target's level. R is measured from the point below the axle at the start of the throw, as Horsdal and his colleagues measure it [74, app. B.1]. The machine swings on without the stone for 1.5 s, so that its loads after release are recorded too.

Friction acts at each pin as a torque equal to the friction coefficient times the pin's radius times the force on the pin [75, eq. 3]: at the axle, and the same form at the hinge. Rolling frames lose C_{rr} times their load on the ground, and the floating arm's counterweight rubs in its guides. The pin forces depend on the accelerations, which depend on the friction, so each evaluation goes round this loop three times. More passes change the range by 4×10^{-6} m (run P6-R02).

Drag on the stone,

acts in the machine as well as in flight, with the pouch counted at the stone's own area. C_d is uncertain by a factor of two or more. Stones of 0.08 to 0.4 m at 20 to 66 m/s have Reynolds numbers of about 10^5 to 2×10^6 , across the drag crisis, where a sphere's drag falls sharply as the air at its surface turns turbulent. A smooth sphere's C_d is about 0.4 up to a Reynolds number of 2×10^5 and about 0.09 by 4×10^5 [76, eq. 1]. Roughness brings the crisis on earlier and holds the drag beyond it at up to about 0.4 [77]. The model uses 0.40 for rough stone. Horsdal assumed 0.25 in one paper, and he and his colleagues about 0.5 in another [74, app. B.2], [75, Sec. 6.7]. Around the hinged optimum, C_d 0.10 gives 698 m and 0.50 gives 550 m, against 580 m at 0.40 (run P7-R05). Like all absolute ranges from the model, these are about 10 to 15% optimistic (Appendix B).

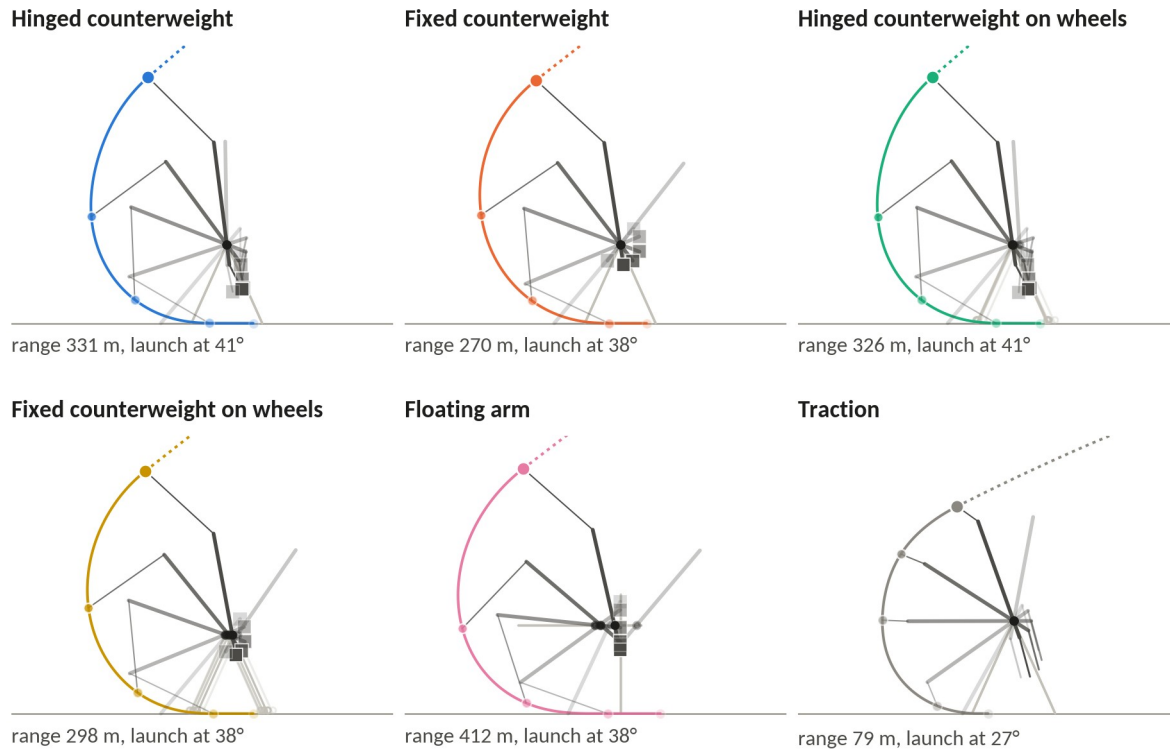


Figure A1. Every layout throws in the same four stages, but the counterweight's path differs, from a circle when it is bolted to the beam to a straight drop in the floating arm's guides. The machines are illustrative, not optimised and not comparable with one another: the five counterweight machines share a 7.2 m beam, 8,000 kg of counterweight and a 60 kg stone, and the traction machine is Tarver's 4.9 m engine, with an assumed 40 kg beam and the crew model fitted to his sixteen pullers and a 4.7 kg ball. Ranges: hinged 331 m (launch 41°), fixed 270 m (38°), hinged on wheels 326 m (41°), fixed on wheels 298 m (38°), floating arm 412 m (38°), traction 79 m (27°). Source: model run P6-R14. Image created by Claude [4].

Scores, Scale and Strength

The paper's scoreboard is range divided by beam length,

R^*_{ideal} is computed in vacuum without friction and with the ideal release, and for given proportions and mass ratios it is the same at every size. R^*_{ref} adds drag, friction, a prong release and a beam sized to its own loads, at the reference scale of Appendix B. It is 58.0 for the Jerusalem 1187 design and 88.5 for the Optimal trebuchet (run P7-R12). Horsdal divides range by the long arm alone [46, eq. 14]. We use the whole beam because the short arm is part of the machine's size and cost. Options are compared at equal M/m , because R^* rises with it, from 20.4 at M/m 25 to 113.9 at 400 for R^*_{ideal} (run P7-R06).

Efficiency compares the stone's energy at release with what the counterweight can give,

where Δh_{cw} is the drop of the counterweight's centre of mass from the cocked position to the lowest point it could reach. Its second term counts only for a weight propped back at ψ_0 . Energy left in the machine at release counts as a loss, and the beam's weight is not netted off, so a heavy long arm is penalised. Two other measures appear only for comparison with the literature. Horsdal's efficiency adds the stone's height at release and nets off the beam's weight [46, eqs. 3, 7–8],

Siano's range efficiency, which Saimre also uses, compares the range with that of an ideal machine putting all the counterweight's energy into a 45° throw in vacuum [49, pp. 66–67], [108, pp. 4–5],

Hansen's machine threw 168 m against an ideal 490 m, an η_R of 0.34 [49, p. 75].

At fixed proportions in vacuum and without friction, a trebuchet twice the size throws twice as far. With lengths measured in units of L_b , masses in units of M and time in units of t , the equations contain no dimensional quantity, so R^* and η depend only on ratios of lengths and masses and on the starting and prong angles. Horsdal's design scaled from a 1.8 m to a 14.7 m beam gives $R^* = 61.31$ at every size, whether its masses grow as the cube or the fourth power of the scale (run P5-R02). His own engines of 1.2 to 10 m share a vacuum $R/(L_1 + L_2)$ of 33.7 [75, table 6].

Drag breaks the rule unless the stone grows with the machine. It enters the scaled equations through one group,

which stays constant only if the stone's diameter keeps pace with the beam. Under full geometric similarity, with C_d held constant, R^* in air is 55.31 at every size. If masses grow faster than the cube of the scale, as under Horsdal's rule for constant beam bending [46, eq. 13], drag matters less on big machines and R^* rises from 52.4 to 56.4 over the same sizes. If they grow more slowly, as for constant shaft strain [75, Sec. 6.3], R^* falls from 56.8 to 54.5 (run P5-R02). Horsdal's engines lose 4.5% of their range to drag at 1.2 m and 7.1% at 10 m [75, table 7].

Strength does not scale. At fixed proportions the bending moment at the axle grows as the fourth power of the scale and the beam's section modulus only as the cube, so the stress rises in proportion to size and a relatively heavier beam is needed to hold it down. The design study therefore compared proportions size-free but chose materials and sized beams at one reference scale.

The beam is sized from the model's own loads. Cutting the arm at a distance s from the axle and balancing the part beyond the cut, with d'Alembert's principle for its weight and inertia, gives the bending moment and the axial force $N(s)$ at each instant. The stress

with Z the section modulus and A the area, must stay below the design strength at every section and every instant (41 sections during optimisation, 161 in the final checks). Appendix B gives the design strengths and the margins behind them.

The arm tapers so that it is equally stressed along its length. Under a load at the tip the moment falls in proportion to the distance from the tip, so a fully stressed solid round arm needs and a diameter proportional to . Such a taper has three fifths of the mass of a uniform arm sized for the moment at the axle, and about 0.37 of its moment of inertia. For the Jerusalem 1187 design, a uniform ash beam sized to the same stress would weigh 321 kg and throw 455 m, against 241 kg and 580 m for the tapered beam (run P7-R13). The same loads size the axle, the hinge pin, the sling and the frame.

Glossary of Engine Terms

The sources name their engines in Greek, Latin, Arabic and the medieval vernaculars, and one word can mean different machines in different centuries. Table A2 defines the terms from what the sources and the historical dictionaries say.

Table A2. Glossary of engine terms, defined from the sources and dictionaries cited.

Term	Language	Meaning in the sources	Sources
Ballista	Latin	For Vitruvius, a stone-thrower built to the weight of the stone it threw. By Ammianus's day, in the fourth century AD, the word meant a bolt-shooter.	[15, 10.11], [109, 23.4.3]
Counterweight trebuchet	Modern English	A trebuchet in which a heavily weighted box replaces the pulling crew.	[24, N. 6]

Term	Language	Meaning in the sources	Sources
Helepolis	Greek, “city-taker”	In Diodorus, Demetrius’s siege tower of nine storeys on four solid wheels, carrying stone-throwers (Salamis, 306 BC). In Byzantine Greek, the traction trebuchet.	[8, pp. 99–100], [110, 20.48.2–3]
Hybrid trebuchet	Modern English	A traction engine with a counterweight added to the short arm, worked by gravity and crew together. Fulton argues, and Purton agrees, that it never existed.	[21, p. 68], [26]
Lithobolos	Greek	Stone-thrower. Polybius’s word for Archimedes’ long-range engines, which the 1923 Loeb translation renders “mangonels”. Anna Komnene lists it beside helepolis and petrobolos.	[8, p. 109], [111, 8.5.2]
Mangonel	English, from Anglo-French <i>mangunel</i> , Medieval Latin <i>manganum</i> and Greek <i>márganon</i>	With petrary, the usual medieval name for traction engines. Attested in Anglo-Norman from 1155. English translations of the Arabic accounts of 1187 call Saladin’s engines mangonels.	[26], [27], [28], [94]
<i>Manjanīq</i>	Arabic, from Byzantine <i>manganikon</i> and Greek <i>manganon</i> , a lever device such as a crane	The larger stone-thrower in medieval Arabic texts, the smaller being the <i>‘arrāda</i> . The Mamluks also used a “Frankish” <i>manjanīq</i> .	[24], [59, pp. 25–26]
Onager	Latin, “wild ass”	Ammianus’s name for the engine once called the scorpion: a single arm, released by knocking out a bolt, that hurled a stone. Wild asses were said to kick stones back at the hunters chasing them.	[109, 23.4.4–7]
Petrary, perrière	Latin <i>petraria</i> , Anglo-Norman <i>perriere</i>	With mangonel, the usual medieval name for traction engines. Chroniclers of the sieges of 1099 and 1191 use it for the crusaders’ engines.	[26], [29], [36, pp. 257–62], [93, p. 205]
Traction trebuchet	Modern English	A trebuchet powered by men pulling ropes on the short arm. Crews of up to 250 threw stones 100 m or more.	[21, p. 68], [24, N. 6]
Trebuchet	Anglo-Norman, 13th century; English from the 15th	A siege engine throwing stones from a sling, from <i>trebucher</i> , to hurl or cast down. Modern historians use it for traction and counterweight engines alike. Purton keeps it for the counterweight engine, which he dates to the 1190s.	[24, N. 6], [26], [30], [31], [112]

The paper uses trebuchet for any beam-and-sling engine, adding traction, counterweight or hybrid when the source of power matters. For Jerusalem in 1187 it keeps the translated sources’ mangonel or engine [94, “Salah al-Din’s Attack”]. No source read says counterweight engines were used there, and Purton, who keeps the word trebuchet for the counterweight engine, dates its appearance to the last decade of the twelfth century [26].

Appendix B. Parameters and Validation

This appendix records the inputs behind the model's numbers and the evidence that the model can be trusted. The model has no flexible parts, so its absolute ranges are about 10 to 15% optimistic at laboratory scale (run P6-R03), while comparisons between designs on equal terms are unaffected.

Reference Scale and Build Contexts

Both designs are sized at one reference scale. The beam is 10 m long ($L_1 + L_2$), inside the 8 to 12 m that Viollet-le-Duc gives for a trébuchet's beam [47, entry "Engin"] and the 10 to 12 m of Finó's working calculation [63, p. 34]. The stone weighs 20 kg, a middle class among the catapult stones excavated at Arsuf [58, p. 87]. The counterweight is 3,000 kg, so M/m is 150, between Hansen's 133 [56, "The Trials"] and the 191 of Horsdal's reference design [73, table 1] (run P7-R01).

The designs are compared in two builds (Table B1). The period build uses what a twelfth-century engineer could make, and the modern build uses today's best materials and bearings. The physical defaults are the same in both.

Table B1. The two build contexts and the defaults they share, as used in the design study. "Assumed" marks a value with no direct source.

Parameter	Period build	Modern build	Basis
Beam	White ash, solid rectangle, tapered	Carbon-fibre tube, wall 3% of the diameter, tapered	Table B2
Axle friction μ_a	0.2, on a 7.5 cm journal (greased iron in hardwood)	0.0015, on 6 cm (rolling bearings)	Greased bronze 0.1 [113, p. 31]; Horsdal's fitted wood-on-wood 0.35 [75, table 1]; ball bearings 0.0010 to 0.0015 [114]
Hinge friction μ_h	0.2, on a 4 cm pin	0.0015, on 3 cm	As for the axle
Ring on prong μ_p	0.15	0.15	Assumed
Hinge or pin load limit	5 Mg (timber box, iron straps)	8 Mg (steel yoke)	Design judgement
Counterweight fill	Stone rubble, 2,000 kg/m ³	Iron or steel, 7,874 kg/m ³	Rubble assumed; iron [115, table 1]
Rolling frame mass m_f	1.0 M	0.5 M	Assumed
Rolling resistance C_{rr}	0.02	0.002 (rails)	Iron rail 0.001 to 0.002 [116, table 1]; 0.02 assumed
Floating-arm	Not available	C_{rr} 0.002, guide	Assumed

Parameter	Period build	Modern build	Basis
rollers and guides		friction μ_g 0.05	
Sling	Hemp, worked at 1/8 of breaking load	HMPE, at 1/5	A rope maker's minimum factor of 5 [117]
Stone drag C_d	0.40	0.40	Rough sphere [77]
Pouch drag in the machine	At the stone's own area	Same	Horsdal uses twice the area [75, Sec. 3.2]
Stone density ρ_s	2,600 kg/m ³	Same	Limestones 1,870 to 2,690 [118, p. 516]; granite 2,690 [119, p. 1]
Air density ρ_a	1.14 kg/m ³	Same	Standard atmosphere [120] at an assumed 750 to 800 m for Jerusalem
Trough friction μ_t	0	0	Neglected in published models [121, p. 448]
Beam drag	Off	Off	A switch in the model
Static friction	1.25 μ	Same	Assumed, for the check that the machine starts

Two defaults carry most of the uncertainty. The drag coefficient of 0.40 is the value for a rough sphere beyond the drag crisis, and Appendix A shows how far range moves if it is wrong. The period axle friction of 0.2 sits between well-greased bronze, about 0.1 [113, p. 31], and the 0.35 that Horsdal fitted to the wooden axle bearing of his laboratory engine [75, table 1]. Hansen greased his hornbeam axles with tallow and beeswax but measured no coefficient [56, "Axles and Bearings"]. The choice matters little at this size: from dry wood on wood to no friction at all, the hinged optimum's range moves only from 553 to 594 m (run P7-R05). The validation runs in Table B3 use each source's own settings instead.

Materials and Design Strengths

Timber strengths in Table B2 are means for small, clear, defect-free specimens, not design values. The US Forest Products Laboratory's Wood Handbook derives a grade's strength from the clear-wood value at its 5% exclusion limit times a strength ratio for knots and slope of grain [81, Ch. 7]. Our design strength follows the same logic. It is 0.375 times the mean, the product of 0.75 for the exclusion limit (assumed until the handbook's variability table is read) and 0.5 for hand-picked logs. The US timber standard allows twice the normal design stress for impact loads [122, table 2.3.2], but that allowance is held in reserve against fatigue over hundreds of shots, which the model does not assess.

Steel and aluminium are worked at half their yield strength, and wrought iron at a quarter of its ultimate strength, the factor Victorian engineers applied to cast iron [123, p. 8]. The carbon-fibre laminate's stiffness and its data-sheet compressive strength of 1,450 MPa [124] are multiplied by 0.8 for off-axis plies, and the strength is then divided by 4. HMPE rope is worked at a fifth of its breaking load, the minimum a rope maker sets [117], and hemp at an eighth. No reliable source gave breaking loads for hemp rope, so the hemp sling was sized from a manila-rope catalogue [125].

Table B2. Key materials, their test values and the design strengths used to size the two designs. Timber values are means for small clear specimens. Blank cells are not needed for sizing.

Material	Density (kg/m ³)	E (GPa)	Test strength (MPa)	Design strength (MPa)	Source
White ash, clear, 12% moisture	672	12.0	106, bending	39.8	[81, table 5-3a]
White oak, clear, 12% moisture	762	12.3	105, bending	39.4	As above
American elm, clear, 12% moisture	560	9.2	81, bending	30.4	As above
Douglas-fir (coast), clear, 12% moisture	538	13.4	85, bending	31.9	As above
Eastern white pine, clear, 12% moisture	392	8.5	59, bending	22.1	As above
Wrought iron, historic	7,700 (assumed)		343, tensile	86	[126, table 1]
Steel S355	7,849	210	355, yield	177.5	[127], [128]
Aluminium 6061-T6	2,700	68.9	276, yield	138	[129]
Carbon-fibre laminate, AS4/8552	1,580	141	2,207, tensile	290	[124], [130, p. 2]
Hemp, single fibres		58	857, tensile	Sling at 1/8 of rope strength	[131]
HMPE rope, 24 mm			Breaks at 388 kN	1/5 of breaking load	[132]
Leather, vegetable-tanned			29.2, tensile	5.8	[133, table 5]

Per kilogram, ash and the straight softwoods make better beams than oak. At the Jerusalem 1187 design's geometry a white pine beam throws 594 m, Douglas-fir 587 m, ash and elm 580 m and white oak 566 m (run P7-R13). Ash was chosen for its toughness under repeated shock and its historical precedent (Appendix D). The counterweight's fill hardly matters: from

earth at 1,330 kg/m³ [134] to lead at 11,350 kg/m³ [115, table 1], the hinged optimum's range moves only from 579 to 582 m (run P7-R05).

Validation

Table B3 sets the model against published models and measured throws. Code checks (S) reproduce another author's calculation under that author's assumptions, and so test the equations and their programming. Measured cases (M) test the physics. The project required at least two independent measured cases to agree within about 15%, which the Aarhus engine and Hansen's machine do. The failures stay in the table.

Table B3. Validation cases. S marks a code check against another author's model and M a comparison with measured throws. Error is the model minus the target, as a percentage of the target. Model ranges for historical machines depend on the assumed beam.

ID	Case	Inputs	Model	Published or measured	Error	Run
V1 (S)	Horsdal's 19.1 t design: range, release, peak bending load at the axle	His geometry and masses; frictionless, vacuum	449.1 m; 66.0 m/s at 39.8°; 1.60 Mg	450 m; 66 m/s at 40°; about 1.6 Mg [73, Secs. 2, 10.2–10.4]	−0.2%; 0.0%; +0.1%	P6-R01
V2 (S)	Horsdal's friction and drag on the Aarhus engine: losses at pivot, hinge and sling; efficiency; vacuum range	Pivot and hinge μ 0.35, sling drag	10.65, 1.85, 0.97 J; 75.5%; 40.1 m	10.7, 1.8, 1.0 J; 75.5%; 40.4 m [75, tables 2–3]	−0.5 to +2.8%; 0.0%; −0.8%	P6-R02
V3 (M)	Aarhus engine in the field, 0.70 kg, 0.87 m sling	Friction as V2, C_d 0.40, tuned release	38.8 m	34.4 $\pm$ 1.5 m [74, table 3]	+13%	P6-R03
V3 (M)	As above, 1.011 kg	As above	29.1 m	24.9 $\pm$ 1.5 m [74, table 3]	+17%	P6-R03
V3 (M)	As above, 0.70 kg, 0.93 m sling	As above	40.7 m	37.5 $\pm$ 1.5 m [74, table 3]	+9%	P6-R03
V4 (M)	Hansen's machine, Nykøbing	2,000 kg ballast, 15 kg ball; beam (450 kg central), box and bearings assumed	180 m; 111 to 253 m over the unknowns	168 m [56, "The Trials"]	+7%	P6-R04
V5 (M)	Saimre's Otepää machine	250 kg, 5 kg stone, from a 35 m hill; beam and most other inputs assumed	48 m; 21 to 108 m	120 m [49, p. 77]	−60% (fail)	P6-R05
V7 (S)	Siano's ladder of range efficiencies	Frictionless; see-saw, then hinge, sling, trough, a	11, 61, 80, 69, 67%	11, 59, 81, 83, 67% [108, table I]	Within 2%, except the trough	P6-R07

ID	Case	Inputs	Model	Published or measured	Error	Run
V11 (M)	Tarver's traction machine	beam of five stone masses Crew model fitted to 16 pullers and 4.7 kg; 40 kg beam assumed	4 pullers: 28 m; 14 pullers: 70 m	52 to 69 m; 145 m [48, p. 162]	rung at -17% -54%; -51% (fail)	P6- R11
V12 (S)	De Jong's four models of Hansen's machine	Frictionless, massless beam, vacuum	278.1, 430.9, 321.6, 68.7 m	278.1, 431.0, 321.6, 68.7 m [78, pp. 12, 19, 26-27]	0.0%	P6- R08
V13 (S)	Levert's fixed and hinged machines, 300 kg beam	Frictionless, vacuum	196.3, 193.2 m; see-saw 28.35 m	196.1, 193.0 m; 28.3 m [79, pp. 25-28]	+0.1 to +0.2%	P6- R09

Four planned cases are missing. Horsdal's optimised designs (V6) became a check on the design study. West's bench engine (V8) could not be set up without the original figure, and Vovard's floating arm (V10) is a different mechanism from ours. Karlsson's wheeled see-saw (V9) rests on a single student report and stays out under the paper's evidence rule, although the model reproduces the gain he describes (run P6-R10).

Two older claims about efficiency also hold. Chevedden and his colleagues found in simulations that hinged machines gave about 70% of their energy to the projectile [21, p. 69], and the model's η_H for five machines has a median of 0.75. Saimre's range efficiency of 0.34 for Hansen's machine [49, p. 75] compares with the model's 0.33 (run P6-R06).

Hansen's machine is the best measured full-size case, with its arms taken from two independent readings of his drawing [49, p. 75], [78, p. 5]. Hansen gave no mass for the beam or the box and no friction for the bearings, so the model runs an envelope of 160 samples across those unknowns, with beams of 250 to 650 kg (Figure B1). With central guesses, including a 450 kg tapered beam, it gives 180 m against the measured 168 m, and 90% of the samples fall between 111 and 253 m. The beam's mass explains most of the spread, which is why Chapter 7 states the beam assumption with every historical model number. One trend is not reproduced. The model gains 49 to 54 m for each extra 500 kg of ballast, where Hansen reports about 30 m [56, "The Trials"] (run P6-R04).

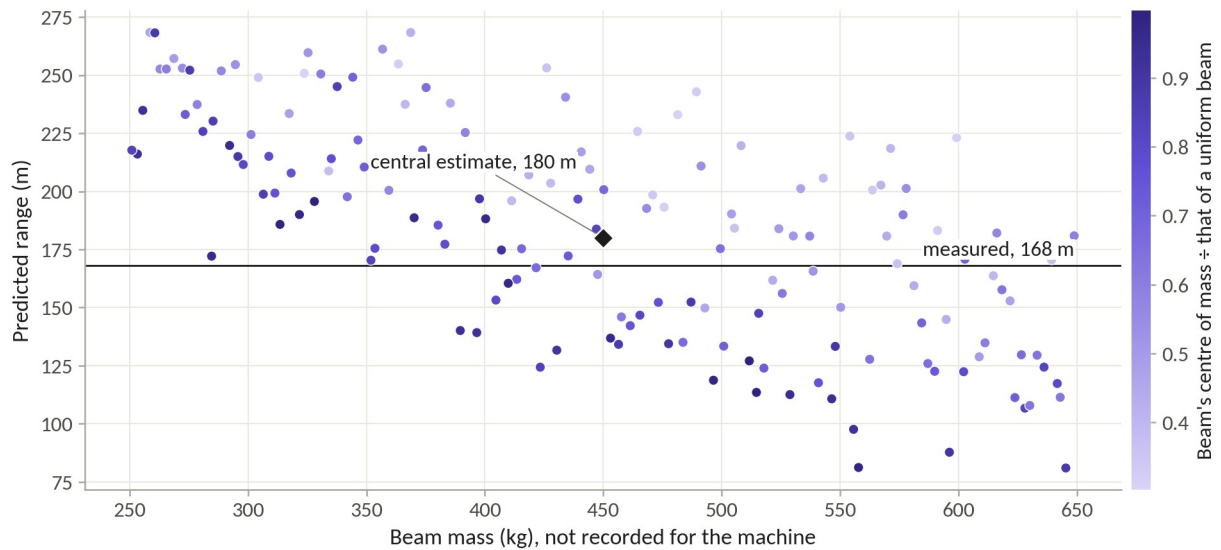


Figure B1. Hansen's measured 168 m lies inside the model's envelope for the inputs nobody published, and the beam's mass drives most of the spread. Each point is one of 160 sampled machines, plotted as range against beam mass and coloured by the position of the beam's centre of mass. The central estimate is 180 m and the middle 90% of samples runs from 111 to 253 m. Source: model run P6-R04, with the measured range from [56]. Image created by Claude [4].

Limits of the Model

The model treats beam, frame and sling as rigid, so the energy a real machine loses as these parts flex and stretch stays in the throw. On his laboratory engine Horsdal measured 33 J of losses where friction and drag account for 13.5 J, and he attributes the rest to these motions [75, table 2, sec. 5.3]. The model's 9 to 17% overshoot on that engine's field throws is about the size of the missing energy, so every absolute range in the paper should be read as about 10 to 15% optimistic (run P6-R03). Every design carries the same bias, so comparisons between them stand.

No measured case exists for the floating arm or for a trebuchet on a rolling frame. At the proportions of a student study by Estes and Estes, the model's floating arm reaches the range efficiency of 0.83 that their abstract reports [71], which is weak support. The margins of these layouts over the hinged machine in Chapter 6 are therefore model results, and they rest on the assumed rolling resistance and guide friction in Table B1 (run P6-R12).

The traction crew model fails outside its calibration. Fitted to Tarver's sixteen pullers throwing 4.7 kg balls 76 to 81 m, with an assumed 40 kg beam, it predicts 28 m for his crew of four, who threw 52 to 69 m, and 70 m for the fourteen who made his record throw of 145 m [48, p. 162] (run P6-R11). Tarver found twelve pullers as effective as twenty [48, p. 158]. Al-

Ṭarsūsī, as Tarver reports him, says that a beam which is flexible, but not excessively so, throws farther [48, p. 149], and a rigid model beam cannot store energy that way. Traction ranges in the paper therefore come from measurements (Appendix C).

Saimre's Otepää reconstruction cannot be reproduced. Its best shot carried a 5 kg stone 120 m with 250 kg of counterweight, from a hilltop about 35 m high [49, p. 77]. With mid values for the inputs he did not publish, the beam among them, the model gives 48 m, and its central machine would need about 480 kg to reach 120 m, close to the 500 kg Saimre had planned [49, p. 76] (run P6-R05). The case is reported as a failure and not used.

The web simulator runs a JavaScript port of the same generated equations. Its ranges match the Python model's to within $9 \times 10^{-7} \%$ on 45 machines covering all six layouts (run P9-R01), and again on 52 machines once the designs and historical machines were added (run P8-R08). The paper quotes no numbers from the simulator.

Appendix C. Historical Dataset

The project's dataset lists 33 engines, from machines in chronicles to a 2025 laboratory model.

Table C1 keeps the nine with a counterweight, a stone and either a range or arm lengths.

Engines known only by their shot or their weights stay out, as do figures resting on one popular source, such as Warwick's and NOVA's. L_1 and L_2 are the long and short arms of the beam, and H is the axle's height. The status column says whether the range was measured in recorded trials or reported by a builder or a second-hand account, then gives the source register's confidence grade, from High for a published trial to Low for a single unchecked claim.

Table C1. Machines with a counterweight, a stone and a range or arm lengths. Blank cells: not published. Visby's best throw was 193 m with an 11 kg stone. ^a Beam length less the short arm (our arithmetic). ^b As printed, and too short to be right (Table C2). ^c Unnamed in the source, probably Hew Kennedy and Richard Barr's machine. The stone was a car. ^d "30-ton", with the ton not defined. ^e Includes some timber. ^f Saimre's estimate by eye. ^g Planned height. ^h Thrown from a hillfort about 35 m above the landing ground. ⁱ Cut down from 6.10 m.

Machine and date	Type	L_1 / L_2 (m)	H (m)	M (kg)	m (kg)	Range (m)	Status	Source
Favé's machine, Vincennes, 1850	part fixed, part hung	10.0 ^a / 0.30 ^b		4,500	11.7	175	reported, Medium	[49, p. 74], [63, pp. 34–35]
Nykøbing Falster (Hansen), 1989	hinged	5.85 / 1.20	3.15	2,000	15	168	measured, Medium	[49, p. 75], [56]
English reconstruction ^c , by 1991	not stated			30,000 ^d	476	80	reported, Low	[21, pp. 67–68]
Visby, 1995	hinged	6.95 / 1.15	3.5	2,650 ^e	15	about 170	builder's figure, Low	[88]
Lihula, 1997	not stated			600	8	100	reported, Medium	[49, p. 75]
Varbola, 1999	not stated	6.35 ^a / 1.15 ^f		1,400	10	120	builders' claim, Low	[49, pp. 75–76]
Otepää (Saimre), 2002	not stated		3.1 ^g	250	5	120 ^h	measured, High	[49, pp. 76–77]
Black Widow (Bertrand), half scale, 2017	hinged	5.64 ⁱ / 1.52	4.57	612	7.26	270	builder's figure, one shot, Low	[89, pp. 4, 9]
Aarhus laboratory engine, 2025	hinged	0.975 / 0.25	0.831	53.9	0.70	34.4 ± 1.5	measured, High	[74, tables 1, 3]

The Machines in the Model

Five machines have enough geometry to run (Table C2). A beam whose mass was never published is taken as 0.20 of the counterweight including its box, the central value for Nykøbing in Appendix B. It is thick at the axle, with its centre of mass at 0.65 and its moment of inertia at 0.70 of a uniform beam's. Black Widow's 181 kg is its builder's estimate [89], and the Aarhus beam was weighed [74, table 1]. Unknown hangers and slings copy the proportions of the Danish machine at Nykøbing, a hanger 1.125 times the short arm and a sling 0.855 times the long arm [49, p. 75], [78, p. 5]. Black Widow's follow Siano's rules, as Bertrand's design did [89, p. 6]. Each run assumes a practised crew's best release.

Table C2. The runnable machines as built and on equal terms. Model ranges assume the stated beam and the best release, and run about 10 to 15% long. Percentages compare the model with the reported range. 5 to 95%: the spread of model ranges as the unpublished inputs (beam, hanger, sling, bearings, drag) vary. R* columns: scaled to a 10 m beam, a 20 kg stone and M/m 150, with the machine's own beam rule or an ash beam sized to its loads. Source: model runs P8-R01 and P8-R02 (Nykøbing's envelope P6-R04, the 1187 design P7-R12).

Machine and stone	Beam	Reported range (m)	Model as built (m)	Model, 5 to 95% (m)	R*, own beam	R*, ash beam
Aarhus laboratory engine, 0.70 kg (M/m 77)	4.86 kg, measured	34.4 ± 1.5	38.8 (+13%)	measured inputs	41.8	57.1
Nykøbing, 15 kg (M/m 147, counting a 200 kg box)	450 kg (0.20 M)	168	180.1 (+7%)	111 to 253	25.8	42.1
Visby, 15 kg (M/m 177)	542 kg (0.20 M)	about 170	162.8 (−4%)	82 to 249	18.9	36.0
Visby, 11 kg (M/m 241)	542 kg (0.20 M)	193	180.7 (−6%)			
Varbola, 10 kg (M/m 140)	286 kg (0.20 M)	120	178.3 (+49%)	91 to 251	24.7	45.1
Black Widow, 7.26 kg (M/m 84)	181 kg, builder's estimate	270	182.2 (−33%)	98 to 195	30.1	50.1
Favé's machine as printed, 11.7 kg	none over 254 kg (391 kg tapered) can start	175	157 at best, with a 184 kg beam			
Jerusalem 1187 trebuchet, 20 kg (M/m 150)	241 kg tapered ash		580		58.0	58.0

Aarhus (+13%), Nykøbing (+7%) and Visby (−4%) agree with the model as built, which overshoots laboratory throws by 10 to 15%. Varbola's claimed 120 m, against 178 m with the rule's 286 kg beam, would need a beam near 400 kg or a poorly tuned release. Black Widow's

single 270 m shot lies beyond all 80 sampled machines, whose beams were within 20% of Bertrand's estimate. As printed, Favé's weights cannot swing a beam heavier than 254 kg, against about 900 kg by our rule, and a short arm of about 1.1 m would reproduce his 175 m. Otepää cannot be reproduced at its reported 250 kg (Appendix B).

Scaled to the reference machine of Chapter 6 (10 m beam, 20 kg stone, M/m 150, period bearings, prong re-tuned), the three medieval-pattern machines (Nykøbing, Visby and Varbola) reach R^* 19 to 26 with their own beams and 36 to 45 with an ash beam sized to their loads. Black Widow reaches 30 and 50, and Aarhus 42 and 57, against 58.0 for the Jerusalem 1187 trebuchet.

Traction engines appear on measured R^* only, because the crew model fits one crew size (Appendix B). Tarver's 4.88 m beam threw 3.1 kg 145 m with fourteen pullers (R^* 29.7) and 4.7 kg 76 to 81 m with sixteen (about 16) [48, pp. 154, 162]. Hjelsvold and McCallum's machines threw 2 to 4 kg about 80 m from arms of up to 6.5 m (about 12) [25]. R^* compares machines only at equal M/m, and a traction engine has no counterweight.

Appendix D. Decision Records

Every feature of the two designs rests on a written decision record, seventeen in all. Table D1 gives each in a line: the precedent, our model's answer with its run, each design's choice and our confidence. The full records, with every source and the range of every option tried, are a companion file (decision_records.md, with a spreadsheet copy). All ranges are the model's and run about 10 to 15% long (Appendix B).

How the Decisions Were Made

Each decision weighed precedent, from reconstructions, treatises and published models, against physics, from the model at the reference scale of Chapter 6: a 10 m beam, a 20 kg stone and 3,000 kg of counterweight (M/m 150). In every run the beam was sized to its own loads, with its depth and taper left to the optimiser and its peak stress held below the design stress of its material. Without that step a notionally light beam flatters every option. The starting machine throws 498 m with a beam of 5% of the counterweight, but 391 m once its ash beam can carry the loads (P7-R01).

The layout, the beam's material and taper, and the counterweight ratio were settled by re-optimising every candidate from scratch (P7-R03, P7-R04, P7-R06, P7-R13). Settings such as the hanger, sling and prong were then swept one at a time around the optimum, with the prong re-tuned and the beam re-sized where the loads changed (P7-R05). Many swept points break a limit, because the hinged optimum sits in a corner: hanger at the ground clearance, frame at its height cap, hinge at its 5 Mg limit (a load equal to the weight of 5 tonnes) and beam at its design stress.

Table D1. The seventeen design decisions, with the choice for the Optimal trebuchet and for the Jerusalem 1187 trebuchet. Model ranges are for the 20 kg stone at the reference scale unless stated, and run about 10 to 15% long. Source: decision records DR-01 to DR-17 and the runs named.

Decision	Precedent	Our physics (run)	Optimal	Jerusalem 1187	Confidence
DR-01 Layout	A hinged weight threw farther than a fixed one [21, p. 69]	Period: hinged 579 m, fixed on wheels 586 m. Modern: floating arm 885 m, next best 747 m (P7-R03, P7-R04)	Floating arm	Hinged counterweight	Medium
DR-02 Base	Hansen's machine	Wheels help a fixed	Axle rolls on	Fixed frame	Medium

Decision	Precedent	Our physics (run)	Optimal	Jerusalem 1187	Confidence
	sat on a heavy fixed base [56]	weight (481 to 586 m) and hurt a hinged one (579 to 560 m) (P7-R03)	a short rail		
DR-03 Hanger	Short arm and hanger reaching the ground give 93.3% efficiency, two-thirds as long 92.2% [46, table 2]	Longest hanger that clears the ground, which puts the hinge at its 5 Mg limit (P7-R03, P7-R05)	Guided drop, no hanger	4.86 m (2.8 L ₂)	High
DR-04 Arm ratio L ₁ /L ₂	Sanudo 5.5 to 6, Saimre 4 to 5 [49, pp. 68, 75]	Broad ridge from 4 to 6; optima 4.8 hinged, 3.1 floating arm (P7-R02, P7-R03, P7-R04)	3.1	4.8	High
DR-05 Beam and axle height	One more metre of axle would add 17% to Hansen's range [49, p. 75]	Taller always wins: axle 7.5 m, 580 m; axle 5.9 m, 432 m (P7-R05)	10 m beam, axle 7.34 m	10 m beam, axle 7.46 m	High
DR-06 Beam mass and shape	A beam five times the stone's mass cuts range efficiency from 83 to 67% [108, p. 20]	Ash sized to its loads: uniform 321 kg, 455 m; tapered 241 kg, 580 m (P7-R13)	Tapered tube, 99 kg	Tapered beam, 241 kg	High
DR-07 Beam material	Al-Ṭarsūsī wanted cherry or a close-grained wood such as cedar [48, p. 149]	Ash 580 m, pine 594 m, oak 566 m; carbon fibre 885 m, steel 640 m (P7-R13)	Carbon-fibre composite	White ash	Medium
DR-08 Bearings	Greased hornbeam axles at Nykøbing [56]	Dry wood 553 m, greased iron 580 m, rolling bearings 594 m (P7-R05)	Rolling bearings	Iron journals in greased hardwood	High
DR-09 Counterweight ratio M/m	About 100 [49, p. 71]; Nykøbing gained about 30 m per 500 kg [56]	Returns fall slowly: 1% more counterweight adds 0.56% of range at M/m 70, 0.37% at 400 (P7-R06)	150 (3,000 kg)	150 (3,000 kg)	Medium
DR-10 Counterweight fill	Villard's chest full of earth [135, p. 195]	Earth to lead changes range by 1% (579 to 582 m) (P7-R05)	Steel block	Rubble box, 1.14 m cube, on a pin	High
DR-11 Sling length L ₄ /L ₁	About as long as the long arm [49, p. 71]	Optimum 1.0; at 0.8, 467 m (P7-R05)	1.1	1.0	High
DR-12 Sling and pouch	Hansen's rope-net pouch lasted about 120 shots [56]	Peak sling pull 56 times the stone's weight (1187), 77 times (Optimal) (P7-	8 mm HMPE rope	25 mm hemp, leather pouch	Medium

Decision	Precedent	Our physics (run)	Optimal	Jerusalem 1187	Confidence
DR-13 Release	Each ring on Hansen's hook cost about 5 m [49, p. 72]	R12) Prong at 35°, 580 m; at 20°, 528 m; at 50°, 521 m (P7-R05)	Steel prong at 32°	Iron prong at 35°, bent to tune	High
DR-14 Trough	A trough allowed a longer sling and more range [21, pp. 67–69]	Without a trough, 530 m against 580 m (P7-R05)	Lined trough	Greased plank trough	High
DR-15 Frame	Favé's frame failed after four shots [49, p. 74]	Peak axle load 4.83 Mg (1187), 7.95 Mg (Optimal) (P7-R12)	Steel portal on a slab	Two oak A-frames, staked	Medium
DR-16 Starting position	About 45° to the horizontal [49, p. 69]; 30° from the vertical [46, Sec. 2]	Set by the frame cap; propping the box 15° adds 2% (P7-R05, P7-R12)	15° from vertical, hanging free	26° from vertical, not propped	High
DR-17 Projectile	Arsuf's stones mostly 8 to 48 kg [58, p. 87]	4 kg flies 791 m, 80 kg 144 m; impact energy flattens past 40 kg (P7-R11)	20 kg stone	20 kg limestone, re-tuned for 8 to 35 kg	High

Where Precedent and Physics Disagree

Mostly the two agree, on the arm ratio, the sling, the trough and the hinged weight. Where they parted we trusted physics if an independent optimisation backed it, and precedent if the model's margin was small or the model offered no natural place to stop.

Physics won on the frame. Low frames kept reconstruction beams far from upright at the start. Hansen's axle was 3.15 m high and his beam began at 32° to the horizontal instead of the intended 45°, the angle Saimre also recommended [49, pp. 69, 75]. The model keeps rewarding height. Horsdal's optimised designs agree, starting 30° from the vertical [46, Sec. 2], and so does Saimre's estimate that one more metre of axle would add 17% to Hansen's range [49, p. 75]. We capped the axle at 0.75 of the beam length, as in the tallest of Horsdal's designs [46, table 2], so the 1187 design starts 26° from the vertical, nearly upright (DR-05, DR-16). The cap is a judgement about what a period crew could build, not a model result.

Precedent won on the counterweight ratio, the timber and the base. At M/m 150 each extra 1% of counterweight still adds about 0.5% of range (Figure D1, P7-R06). Saimre advised a ratio of about 100 [49, p. 71]. We kept 150, inside the reconstructions' band (Hansen's 133, Visby's 177), because 3,000 kg of rubble fills a 1.1 m box that a crew can load and haul down.

Pine and Douglas fir beat ash by 1 to 2% in the model, but ash, the wood of the documented reconstruction arms, was kept for its toughness under repeated shock [56], [88]. A fixed weight on a rolling frame ties with the hinged machine (586 against 579 m), but only on an assumed rolling resistance and a prepared track. The 1187 design keeps a fixed frame (DR-02), as Hansen's machine had [56].

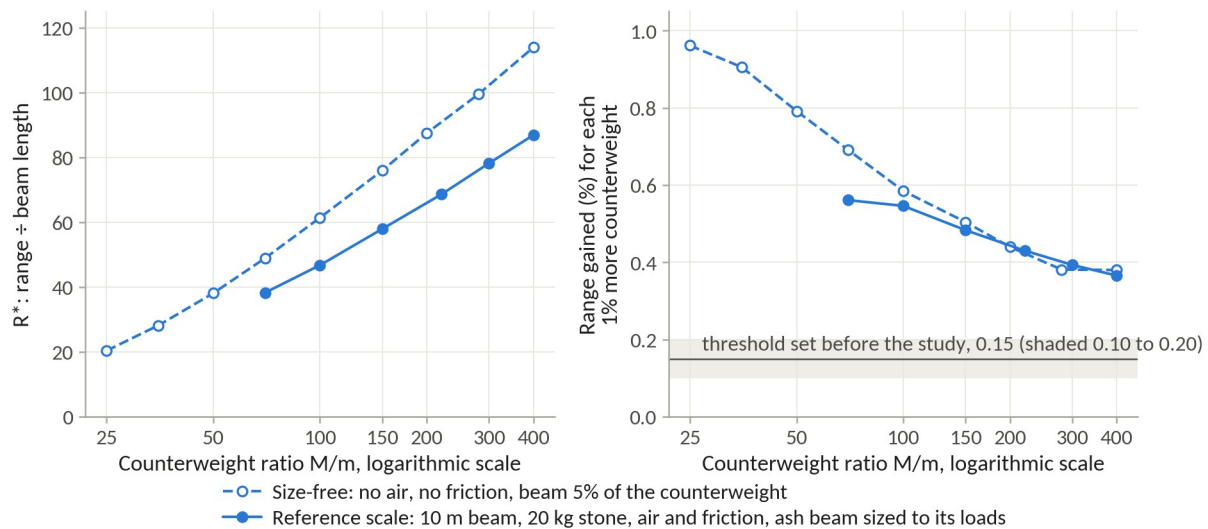


Figure D1. Range keeps rising with the counterweight, and the return on extra weight falls only slowly: at M/m 150 each 1% more counterweight still adds about 0.5% of range, far above the 0.15% below which extra counterweight was to be judged not worth adding. Left, R^* against M/m . Right, the range gained for each 1% more counterweight. Dashed: no air, no friction, a beam of 5% of the counterweight. Solid: the reference machine with an ash beam sized to its loads. Source: model run P7-R06. Image created by Claude [4].

Propping is the one period claim the model does not support. Chevedden and his co-authors wrote that propping the box up gave it farther to fall [21, pp. 68–69]. In the model, holding the hanger back 15° adds 2% and 45° costs 5% (P7-R05), so the 1187 design's box hangs free.

What Building Errors Cost

A design that works only when built exactly would be little use in a siege camp. Figure D2 tests both designs against a 5% change in each input, and against 1,000 machines with every length and mass up to 5% off. The counterweight, the short arm, the stone's mass and the sling matter most. The median loss is 1.0% for the Jerusalem 1187 trebuchet and 0.6% for the Optimal, and one machine in twenty loses at least 6.8% (Jerusalem 1187) or 5.8% (Optimal) (P7-R10, P7-R08).

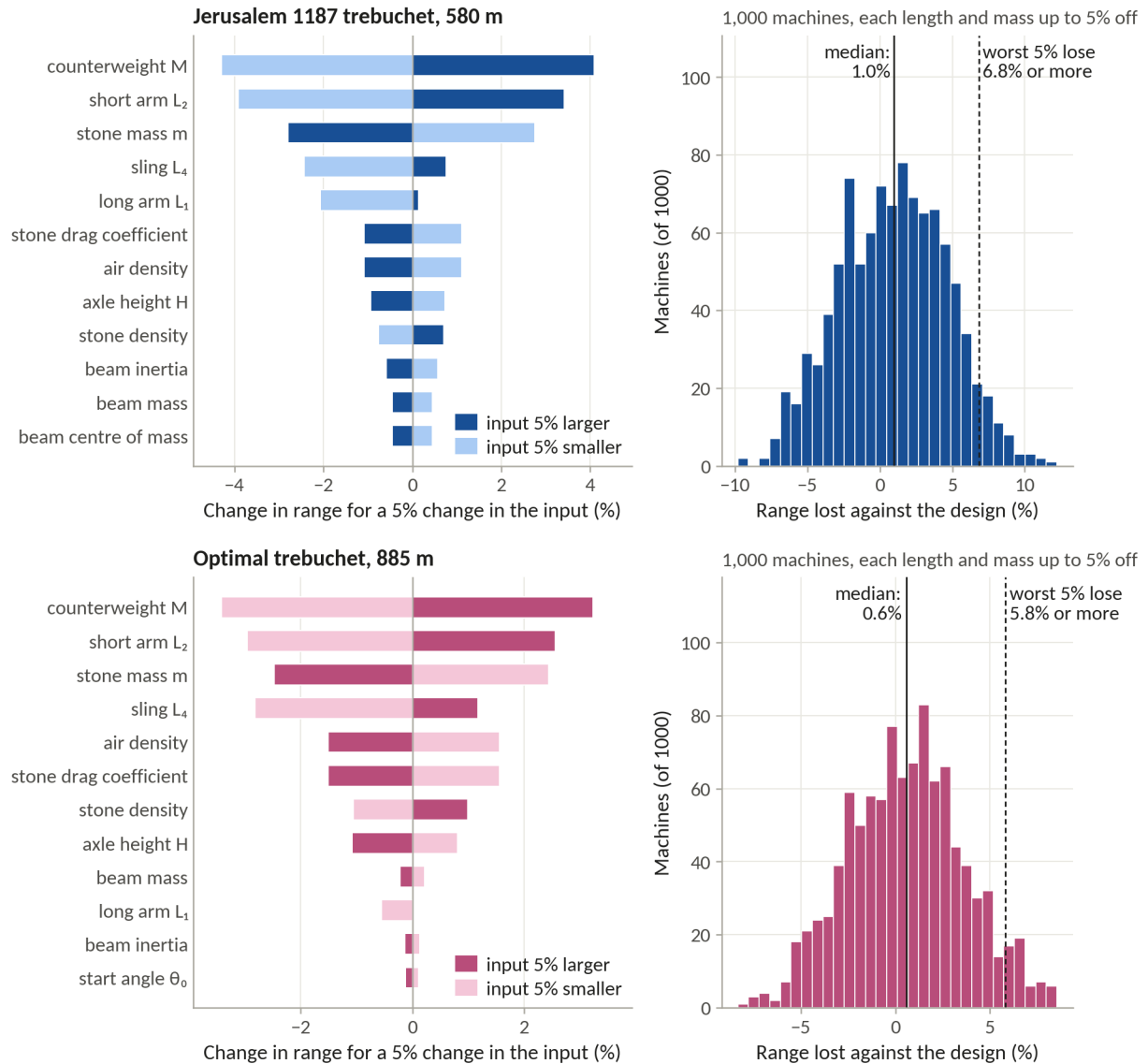


Figure D2. Both designs tolerate ordinary building errors: with every length and mass up to 5% off, the Jerusalem 1187 trebuchet loses a median 1.0% of its range and the Optimal 0.6%. Left, the change in range for a 5% change in each of the twelve most influential inputs. Right, 1,000 machines built with random errors, with the median and the worst 5% marked. Source: model runs P7-R10 (Jerusalem 1187) and P7-R08 (Optimal). Image created by Claude [4].

Appendix E. Simulator Guide

The Trebuchet Proving Ground is a web page that runs the same equations as the model behind this paper. It was built with Claude [4] from the project's model and cross-checked against it. Readers can change any part of a machine and watch it throw, drawn to scale, beside up to two others. The page is at <https://claude.ai/artifact/MT4E2gsdMQyxrks1mVcWfk> and opens for readers once the author has shared it. A standalone copy (simulator.html) is among the companion files and runs in any browser.

What Can Be Changed and What It Shows

More than 70 inputs, each with a slider and a number box, sit in twelve groups running from the proportions to the air. Switches change the layout between counterweight, floating arm and traction, and add a hinge or wheels. The stone can leave at the best moment a crew could find, off a prong at a set angle, or at a chosen climb angle, beam angle or time.

The throw plays back at down to a twenty-fifth of real speed. Readouts give the range, R^* , the efficiency, the release speed and angle, the impact energy and the peak loads on the axle, hinge and sling, with an energy check on every throw. When a machine fails to lift or throws backwards, the page says so in plain words. Tabs plot the flight path, energy, angles and loads, sweep any one input across a range, map 143 throws by arm and sling ratio, and compare machines in a table.

Presets

Nineteen presets load ready-made machines (Table E1). Each has a note giving its source and status and, for the historical machines, the beam assumed.

Table E1. The simulator's presets. The design and scenario presets reproduce runs P7-R12 and P8-R05, the historical machines run P8-R01.

Group	Presets	What they show
The two designs	Optimal trebuchet; Jerusalem 1187 trebuchet	The designs of Chapter 6. The 1187 preset draws the north-east wall at a 100 m stand-off (main wall 11 m, a proxy) and uses thinner air for Jerusalem's height (1.14 kg/m^3 , assumed)
Jerusalem 1187 scenario	108 kg stone at the stand-off; 32 kg stone from beyond crossbow range (421 m)	The 1187 design with sling and prong re-tuned for each stone, as in Chapter 8

Group	Presets	What they show
Historical machines	Nykøbing; Visby; Varbola; Black Widow; Otepää; Aarhus laboratory engine	The machines of Appendix C, each note stating its beam: 0.20 of the counterweight, Bertrand's 181 kg, or measured
Historical types	Traction (Tarver, 16 pullers); fixed counterweight; hinged counterweight	Demonstration machines, not optimised
Modern types	Hinged and fixed counterweight on wheels; floating arm	Demonstration machines. No measured case exists for wheels or the floating arm
Published models	Horsdal's 19.1 t design; Levert's hinged and fixed machines	Published ranges the model reproduces (Appendix B)

Reproducing Figures 6, 8 and 12

The page shows the ideas behind Figures 6 and 8 (Chapter 6) and Figure 12 (Chapter 8), though not their exact numbers, because it has no optimiser and a sweep leaves the sling and prong untouched. It gives no numbers to quote: every number in the paper comes from a model run with a run ID.

For Figure 6, load the Optimal trebuchet, add a machine with “+ Compare a machine”, load the Jerusalem 1187 trebuchet into it and open the Compare tab. Switching off the 1187 design's hinge, or putting it on wheels, shows why layouts differ. The changed machine is not re-optimised, though, so it will not match its bar in Figure 6 (P7-R03, P7-R04).

For Figure 8, load both designs and sweep the stone's mass from 4 to 80 kg, plotting the range and then the impact energy. Heavier stones fly shorter, and the energy levels off past about 40 kg. The page's curves fall below the figure's away from 20 kg, because Figure 8 re-tunes the sling and prong for every stone (P7-R11).

For Figure 12, load the Jerusalem 1187 trebuchet, which draws the wall 100 m away, and add the two Jerusalem scenario presets as further machines. The Trajectory tab draws all three throws against the wall. Each scenario preset moves the wall to its own target, and the Scene group sets it wherever the reader likes. A late, flat shot can be set with the release mode “when the beam reaches an angle”. Figure 12's scatter of 200 shots per mode (P8-R06) is not in the page.

Agreement with the Model and Limits

The page's physics is a line-by-line port of the Python model, checked against it on 45 machines (P9-R01) and again on 52 after the designs, the historical machines and the Jerusalem settings

were added (P8-R08). All passed the 2% mark with room to spare. The largest range difference was $8.8 \times 10^{-7} \%$, under a millionth of a per cent, at the page's own tolerance, and $6 \times 10^{-4} \%$ at the looser tolerance used for sweeps and the map. Release speed, efficiency, impact energy and peak loads agree to within $3 \times 10^{-5} \%$. The two programs use different numerical solvers, so this tests the equations and the release logic, not one headline number.

The page shares the model's limits. Beam, frame and sling are rigid, so ranges run about 10 to 15% long against laboratory throws (P6-R03). The traction crew is fitted to one crew size (P6-R11), and the stone's drag is uncertain near the drag crisis, where the airflow round a ball changes. The page reports loads but sizes no parts, so a machine that throws far on screen may not survive in timber.

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